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*Title:*
**Essays in the theory of organizational structure.**

# Essays in the Theory of Organizational Structure

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A thesis submitted to the University of Bristol in accordance with the requirements of the degree of Doctor of Philosophy in the Faculty of Social Sciences, Department of Economics.

November 1999

Word count: 56,582

## Abstract

Organizations are complex structures of interacting principal-agent relationships, and the incentives created in these structures are not always clear. Our aim is to clarify some of these relationships.

A large class of problems in the field of organizational design possess a common structure: agents who may own private information are subject to incentives provision from several principals. This structure is known as *common agency.*

Chapters 2–4 review the existing theoretical literature on common agency, and outline the common structure of the available models under symmetric and asymmetric (moral hazard and adverse selection) information. The aim of this review is to begin consolidation of this literature, and to point up directions for future research. Subsequent chapters take up some of these directions in applications.

Chapter 5 studies incentives provision in a hierarchy in the context of state procurement of assets necessary to provide a public service. We find that giving asset ownership to a private service provider may improve the quality of investment and therefore the quality of service provision. We conclude that the state should seek greater private sector involvement in asset procurement.

Chapter 6 takes up the issue of common sales agency and considers sales incentive (commission) payments to common agents as potential signals of unobservable (to the consumer) product quality, an aspect the literature on common sales agency has generally ignored. We find conditions under which sales-based compensation leads to optimal information transmission.

Finally, in chapter 7 we study the interaction of incentives between two antitrust agencies who regulate a common monopolist-agent. In contrast to the generic common agency case, we model the principle of subsidiarity: one agency has greater power than another. Generally, this results in possible inefficiencies in the regulators' information acquisition about firm conduct. We derive limiting results about the optimal behavior of the regulatory agencies.

## Acknowledgements

Every author receives help in the preparation of their work, and often those sources are too numerous to mention. I have tried to be methodical about the papers that have helped me understand the class of problems that this dissertation addresses. But memory fails to recall all the conversations with other economists that have often shaped my ideas. Most importantly, however, I have the pleasure to acknowledge the useful advice, constructive criticism, and help I have had from colleagues and friends at the University of Bristol, and at Dartmouth College. Their help and encouragement has been a constant source of inspiration. Amongst them, two people stand out: above all, thanks are due to my advisor, Professor Ian Jewitt, for his patience, his ability to motivate and incite, and for his help and support with more than just this dissertation; and to Professor Paul Grout, who has given so freely of his time and ideas, all of which are alas! only imperfectly reflected in this dissertation.

Certain chapters have benefitted more than others from the advice of certain individuals. Chapter 5 from the extraordinary help of both my co-authors, chapter 6 from Isaac Alfon's comments, and chapter 7 from Professor John Scott's criticisms and advice.

Finally, thanks are due to those institutions that have provided financial support during the writing of this dissertation. These are the Department of Economics at the University of Bristol, for giving me the pleasure and responsibility to teach some outstanding individuals; University of Bristol Convocation, for three-year funding through their scholarship scheme; the Association of British Insurers, for a one-year scholarship; and finally Dartmouth College for its hospitality and inspiration during the final months of the preparation of this dissertation.

Although I have been given help so generously, there will be some remaining omissions and errors. For these, the standard disclaimer applies: they are of course my responsibility alone.

Lastly, two people have always believed in me, and on occasion have had to help me believe in myself: they are my parents. To them I dedicate this dissertation.

## Author's Declaration

I declare that the work in this dissertation was carried out in accordance with the Regulations of the University of Bristol. The work is original except where indicated by special reference in the text. No part of the dissertation has been submitted for any other degree or examination, either at this or any other university.

Any views expressed in the dissertation are those of the author and in no way represent those of the University of Bristol.

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# Chapter 1

# Introduction

Contracting relationships are a ubiquitous fact of economic life. Whenever decisions are delegated, there is scope for the objectives of the decision-maker and the center to differ in a way that requires comprehensive contracting between decision-maker and center. Typically, organizations are complex webs of such contracting relationships. In fact, we wish to interpret the term "organization" broadly here, and use it to refer to any structure in which decisions are taken in a decentralized way. Within most, if not all organizations, decision-makers have some degree of autonomy to pursue their own goals rather than those of the organization, or the goals of those on whose behalf they take decisions. In general, one should therefore expect that decentralized decision-making does not result in optimal decisions for the group of agents within the organization.

Decentralized decision-making of itself, however, need not imply the presence of inefficiency. In fact, trade in complete markets (which is just a decentralized decision-making process) is typically efficient given well-known conditions: The presence of complete markets ensures the existence of prices that internalize any inefficiencies costlessly. The contracting problem therefore assumes real economic interest only when the objectives of decision-maker and center cannot costlessly be aligned. The explanation most commonly offered for the necessity of costly alignment is an informational asymmetry between the contracting agents.[1,2] Typically, we assume that the decision-maker has better information than the center. This is natural: presumably, decision-making is decentralized precisely because the center is constrained (whether informationally or otherwise) in its ability to take decisions itself, or in its ability to monitor the decision-maker. When information is asymmetrically distributed, the center will need to provide incentives for decision-makers to reveal their private information (or to act in the center's interest). These incentives are generally inefficient, in the sense that decision-makers can use their private information to extract rent from the center. Since the center dislikes costly rent, incentives are lower powered than is (first-best) optimal.

We now have a good understanding of the issues arising from asymmetric information in the relationship between decision-maker and center when there is one decision-maker and one center (cf. chapters 3 and 4 and the references cited therein), and when there is one center who seeks to control several decision-makers (for instance Holmstrom (1982)). While it has been classical to study monetary incentives, more recent work has focussed on the provision of incentives through the career concerns of decision-making units, and through mission-setting for organizations (cf. Dewatripont, Jewitt, and Tirole (1999a), Dewatripont, Jewitt, and Tirole (1999b)).

However, not all contracting relationships fit this two-player mould. In large organizations, several other structures are common. Relationships between decision-makers and center may be (multi-level) hierarchical (cf. Tirole (1986)), or one decision-maker may have decision-making power delegated to her from several individuals with potentially different objectives. Although there is now a growing literature on these structures, our understanding of how incentives are provided, and what the implications for the outcomes of decentralized decision-making are, is still limited. The aim of the following chapters is to

[1]Take, for instance, a moral hazard setting with hidden action. If the agent's actions were observable (or, in a setting with uncertainty, could be inferred) and the principal risk neutral, the principal would bear all risk Spence and Zeckhauser (1971). More generally, it can also be shown that, even with a risk-averse principal, risk-sharing is optimal. Furthermore, in an adverse selection setting with hidden information, when the agent's type is observable, the contracting problem disappears.

[2]There is a large literature on why certain transactions are carried out inside firms rather than through market interaction. The literature begins with the seminal paper by Coase (1937). Important contributions are Williamson (1985), and the recent literature on contractual incompleteness (Grossman and Hart (1986), Hart (1995)). This literature, however, is largely independent of our concerns in this dissertation: We focus on incentive provision within a given structure, without the need to explain why that structure is chosen. Chapter 5 is an important exception: there we discuss why a certain organizational structure should be chosen over another.

contribute to our understanding of these structures.

The purpose of the first part of this introduction is to lay out a rough structure as the basis for the following discussion. Section 1.1 therefore classifies models of asymmetric information. Section 1.2 clarifies the principal-agent paradigm, which simplifies considerably the complex bargaining process under asymmetric information that contracting would generally give rise to. The section also raises questions about the standard single-principal single-agent framework that lead to a discussion of common agency.

## 1.1 Classificatory Remarks

Models of contracting under asymmetric information may usefully be classified along two axes: according to the timing of the informational asymmetry (does it arise before or after contracting?), and according to the nature of the private information.

The asymmetry may arise before contracting (the parties' trading decisions depend on it), or it may arise under a contract (the parties' performance under the contract depends on it). Accordingly, *adverse selection* models are models of pre-contractual private information; *moral hazard* models are models of post-contractual private information.

Either type of model can be further divided according to the informational structure of the model: the uninformed party may be unable to observe the actions of the informed party; or the uninformed party may be unable to observe some characteristic ("type") of the informed party (or both). *Hidden action* models are models in which the informed party takes unobservable actions linked to observable outcomes via a stochastic technology; the informed party's problem is to induce the appropriate action through the design of a contract contingent on observable (and verifiable) outcomes. *Hidden information* models are models in which the informed party possesses private information about some characteristic (her "type"); the informed party's problem is the design of a contract that induces revelation of the informed party's type.

The literature has not always made this distinction clearly. Sometimes adverse selection and hidden information are equated, as are moral hazard and hidden action. This, of course, is misleading: both moral hazard and adverse selection models may each be combined with any informational structure. Since the majority of the technical literature does however analyze adverse selection models of hidden information and moral hazard models of hidden action, our nomenclature will occasionally follow this convention. Wherever confusion may arise, we will make precise the nature of the model and its informational structure.

## 1.2 Principal-Agent Analyses

As mentioned before, contracting under asymmetric information gives rise to a complex bargaining problem. To make the problem tractable, the principal-agent paradigm assumes that one individual (the principal) can make a take-it-or-leave-it contractual offer to the other (the agent). In effect, the principal designs a mechanism (or, more generally, game) to be played by the agent. If the agent declines, the interaction is terminated.[3] The model has been applied to the design of incentives, both for individuals and teams; to the design of optimal taxation schemes; nonlinear pricing schedules; etc.

### 1.2.1 Common Agency

Some contractual relations, however, are poorly captured by the standard bilateral principal-agent paradigm. For instance, multiple producers may use a common retailer (or marketing agency) who possesses private information about her actions (hidden action), or about market conditions, the identity or preferences of consumers, etc. (hidden information).[4] Or consider the following example drawn from the internal organization of government: a firm that possesses private information about its cost parameters is subject to regulatory regimes from multiple regulatory agencies, each of which aims to extract information about the firm's cost parameter.[5] Or, in an example from the literature on tax competition, different legislations may design tax regimes in competition over the location of a large taxpayer (whose productivity is privately known).

In situations such as these, when principals cannot co-operate (for instance, through lack of coordination between different regulatory bodies, or because coordination between competitors is prevented under competition law), the analysis must take into account the

[3] The principal-agent terminology follows Ross (1973).

[4] cf. Bernheim and Whinston (1985), Bernheim and Whinston (1998), Martimort (1992), Martimort (1996a)

[5] cf. Martimort (1996b)

fact that multiple principals are non-cooperatively setting contracts for a single agent.

The literature on common agency (Bernheim and Whinston (1986a), Martimort (1992), Stole (1992)) therefore views complex organizations as networks of bilateral relationships between one of a number of principals and a single agent, each ruled by a bilateral contract. The analysis focuses on the description of the equilibrium of the non-cooperative game of contract-setting between principals. In general, one should expect the equilibrium outcome to differ both from the symmetric information outcome and the single-principal single-agent contracting outcome. We know that inefficiencies are already present in the single-principal single-agent model. Since the agent may "lie" (that is, not reveal, or misrepresent her private information), the principal needs to leave costly rent to the agent as an incentive for truth-telling. But this rent alters the relative prices of decisions, so that generally the decision that is taken by the agent in equilibrium will be suboptimal.

In addition, when several principals non-cooperatively design incentive schemes (or, mechanisms) for a single agent, externalities will typically arise. Since one principal has to allow the agent to obtain rent (in order to obtain the agent's private information), competing principals can extract some of this rent from the agent. This externality (externality in rent extraction, or "type 1" externality, cf. Laffont and Martimort (1997)) is well documented in the theoretical common agency literature. We will sometimes refer to this externality by its more descriptive name and call it an "indirect contractual externality" because it arises indirectly through the agent's utility function (which determines rent). Another type of externality ("type 2") arises more straightforwardly when principals have opposing preferences, and attempt to influence the agent to take decisions favorable to them. This externality is standard: each principal wishes the agent to take a decision in her favor, but if the decision the agent takes on behalf of principal 1 is correlated with the decision the agent takes on behalf of principal 2, the two principals will impose externalities on each other. We sometimes refer to this as a "direct contractual externality" because it arises directly through the opposing preferences of the principals. Again, in equilibrium, the action will be inefficient.

### 1.2.2 Common Agency with Symmetric Information

But it is important to note that inefficiencies do not only arise in common agency models when information is asymmetrically distributed. Principals impose externalities on each other when they influence a common agent's actions even under symmetric, and perfect and complete, information (Bernheim and Whinston (1986b), Dixit, Grossman, and Helpman (1997)). The non-cooperative bidding of principals for decisions taken by the agent (where the decision enters each principal's payoff function) leads to externalities in just the usual way: If principal preferences are not perfectly aligned, there will typically be an incentive for one principal to bring about an agent decision that is less favorable to a competing principal, and depending on the nature of the externalities, incentive provision is likely to be too high or too low. This is just what we called a "type 2" or "direct contractual" externality, above.

### 1.2.3 Intrinsic and Delegated Common Agency

A final classificatory remark on common agency models is due. Depending on the context, one might want to model the situation as one of intrinsic common agency (the agent can either participate in the mechanisms of all or none of the principals), or one of delegated common agency (the agent can choose any subset of principals in whose mechanisms she wants to participate).

Typically regulatory contexts, where a single firm is regulated by several independent regulators, make an intrinsic common agency formulation appropriate: if the firm chooses to produce, it is subject to regulation from all principals. Of course, the firm can always choose not to produce at all, in which case it can avoid regulation altogether. In some contexts, however, a delegated common agency model seems more appropriate. A common sales agent typically chooses the manufacturers whose products she sells: for instance, an independent financial adviser would typically choose the insurers whose products she sells.

A very general model should contain a general specification of these constraints in the agent's participation constraint. In most cases, however, it turns out that the intrinsic agency formulation is informative (and certainly simpler). Delegated common agency would require that for each principal separately, the agent's participation constraint is fulfilled. While this should not materially alter the approach we take, certain results (such as the result in chapter 3 that, under joint observations of the agent's outputs in two principals' mechanisms, principals use the common agent to effect transfers between them; that is, while principal 1 pays the agent, the agent pays principal 2) would disappear: the agent could simply refuse to contract with one of the principals. In general, the literature has focused on intrinsic common agency models (an exception is the paper by Bernheim and Whinston (1986a) which contains a general specification of the agent's participation constraint that is broad enough to encompass the two model types).

## 1.3 An Organizing Example

To fix ideas, and to motivate the discussion of common agency models, consider the following organizing example of a common agency situation. In a little-known paper, Coase (1979) studies a common agency problem in radio broadcasting. Several record producers (principals) simultaneously design mechanisms for a common agent (the radio D.J.) in order to induce the D.J. to include specific programming content (the record producer's record) in a broadcast program. This practice has come to be known as "payola." The mechanisms tend to be of a simple affine form: fixed payments are made each time a record is played. Because of its allegedly undesirable consequences for program content, payola is illegal in the US since the 1960 amendments to the Communications Act, and specifically excluded in UK broadcasting licenses.

Coase however conjectures that payola may have no undesirable welfare consequences: his argument, briefly, is that allowing the D.J. to obtain payola payments for the records she plays will not alter program content. Record producers would be willing to effect transfers to the D.J. up to the increase in profits that increased sales could achieve. But audience preferences for listening to, and owning, records presumably exhibit a similar ranking of records, so that records more preferred by the audience would be bought more frequently and accordingly be associated with larger payola payments. Furthermore, Coase argues, the D.J.'s payoff function is directly affected also by audience size (for instance, because the D.J. employment prospects depend on it), so that the incentive to deviate from the program content that maximizes audience utility is weakened further. In an informational twist on the argument, Coase conjectures that the D.J. has better information about audience preferences than a radio station owner, or possibly even the record producer, so that using payola payments to elicit this information may be an important information transmission mechanism.

We take up several possible interpretations of this story to motivate our discussion of the theoretical foundations of the common agency literature. First, when informational asymmetries are absent, payola payments may be a pure incentive payment to play one record producer's record over another's. This opens up the literature on common agency under symmetric (perfect and complete) information, which we review in chapter 2. An alternative interpretation is of the Coasean example as one of common agency under conditions of moral hazard: suppose, for instance, that monitoring the D.J.'s actions is costly (activities that cannot be monitored easily might be remarks, favorable or unfavorable, about the quality of a record that is about to be played, remarks about the artist signed to a record producer, etc.). When actions are unobservable, payola payments may be a simple incentive mechanism to induce the D.J. to take the action preferred by the record producer, or to increase "sales" effort for that producer's record. We review the relevant theoretical literature on common agency under moral hazard with hidden action in chapter 3. A final, and different, interpretation of the story is as one of contracting under asymmetric information: the D.J., for instance, may possess better information about market conditions, consumer preferences, or suitability of a record for her broadcasting program than the information held by record producers. On this interpretation, payola payments are a mechanism to elicit this information from the D.J. The literature on common agency with adverse selection and hidden information is surveyed in chapter 4.

## 1.4 Overview

The thesis falls roughly into two parts. As already noted, the first of these (chapters 2–4) is methodological. These chapters survey the literature on common agency with a heavy bias toward technique rather than application. Each of the chapters (chapter 2 for common agency under symmetric information, chapter 3 for common agency under moral hazard, and chapter 4 for common agency models under adverse selection), however, contains a section reviewing some of a variety of applications of the theoretical model.

The second part (chapters 5–7) takes its cue from the discussion in chapters 2–4 and contains the main modelling effort of the dissertation. The following is a brief guide to these chapters.

### 1.4.1 Hierarchies in Public Service Provision

Chapter 5 addresses the question of how to design a hierarchy in which information is distributed asymmetrically. From a modelling perspective, the chapter is a hierarchical, not a common agency, model. However, in a later chapter (chapter 7) we study a common agency problem within a hierarchical structure; and the two model types are of course closely linked. The attractiveness of this chapter is the application it studies: we analyze the implications of different organizational choices (in this case, ownership structures) on the incentives for cost reduction and quality of service provision of a public service.

In many countries, the role of the state has shifted from that of a provider of public services and owner of assets for the production of these services, to that of a designer of mechanisms for the private provision of services and private ownership of capital assets. In particular, in the UK the Private Finance Initiative (PFI) seeks to increase private sector involvement in the building of assets required for the provision of public services. Projects under PFI are as diverse as schools, hospitals, prisons, mail sorting facilities, etc. Despite the heavy political emphasis placed on PFI and other projects pursued as public-private partnerships, we know of no argument for the effectiveness of private rather than public procurement of assets when complete contracts can be written.[6] We provide such an argument.

We study the case in which a builder has the option to invest in (unobservable) cost reduction (or asset quality) of an asset necessary to provide a public service. The builder's investment determines the cost of service provision by a (private) service provider. Ideally, the principal (the state) would wish to write a complete contract with the service provider that is fully revealing of cost conditions. Given this information about cost, an incentive

[6]The literature on contractual incompleteness (cf. Grossman and Hart (1986), Hart (1995) and the references cited therein) has addressed the issue of how different ownership structures create different incentives when complete contracts cannot be written. Our concern, however, is with the implications of different organizational choices on incentives when complete contracts are possible. We therefore address a different set of issues.

scheme for the builder (conditional on the cost realization) may be designed; typically such an incentive scheme consists of a damage contract, enforceable in court. The central insight of our model is that when the principal owns the asset (that is, buys it from the builder), she can claim damages from the builder, conditional on the observed cost realizations that emerge from the revealing contract with the service provider. In this case however, there is an incentive for the principal not to obtain information about costs, so that she can credibly claim damages from the builder whether the investment has been made or not. This, in turn, destroys the builder's incentives for investment.

We show that a change in the organizational structure of public service provision, towards greater involvement of the private sector (that is, the private service provider owns and builds the asset) may lead to greater investment incentives by eliminating this commitment problem. In fact, we obtain the somewhat nonstandard conclusion that assets should be owned (and built) privately precisely when the public service that is to be provided with that asset is essential (in the sense that demand for the service is inelastic). Finally, we endogenize the organizational design choice. Presumably, the state can choose the structure of ownership of the asset used to provide the public service. We obtain conditions for the optimality of public and private asset ownership. These conditions give a fuller answer to the question: "What should the state buy?"

### 1.4.2 Common Sales Agents

Much of the applied literature on common agency has studied common sales agency: a single sales agent retails products from different manufacturers (principals). Variously it has been assumed that the agent owns private information about a market or customer characteristic (as in the adverse selection context), or about her sales effort. In each case, the agent's reward for selling a principal's output acts as a mechanism for information revelation or as an incentive scheme to take the appropriate (desired by the principal) sales action. The general conclusion from the literature on common agency under asymmetric information is one of inefficiency: compared to second-best (when principals can coordinate actions), the agent's decision is distorted. This general sense of inefficiency has often led commentators to conclude that sales-based compensation for retailers (such as insurance brokers, travel agents, or radio disk jockeys in the Coasean example) distorts the mix of products these retailers sell, or the sales advice that they give to their clients.

However, one must be careful not to overstate this aspect of the problem: a different mechanism may be at work at the same time. It may be true that differential sales-based payments from several principals leads the common agent to sell more of the good that attracts the higher (incentive) payment. But if selling goods of a higher quality gives the sales agent greater incentive payments, there is no distortion away from the optimum. This, in fact, is one interpretation of the Coasean suggestion that (in the context of radio programming), output-based rewards may not lead to a distortion away from the optimum.

The story, of course, is familiar in a different context. In the well-known signaling model of Spence (1974), higher types (with private information about their characteristic) are able to differentiate themselves by undertaking costly, but observable, actions. When cost is related to type, an equilibrium may result in which observable actions are revealing of type.

Chapter 6 makes this connection. The chapter studies common sales agency for investment products. In fact, we motivate the model by appeal to the controversy over sales-based commission payments from providers of life assurance savings products to independent financial advisers. We study different assumptions about the behavior of investment products, and find a general welfare result: sales-based commission payments, under the assumptions placed on the behavior of the financial product, is generally welfare-improving for consumers of these products.[7]

Similar conclusions, of course, hold for other types of sales-based retailer compensation, and we would, in general, wish to caution against an over-literal interpretation of the inefficiency results from the theoretical common agency literature when signaling is an important part of the structure of the problem.

[7] One of the important assumptions is that "quality" is one-dimensional, and there exists a clearly defined "better" and "worse," that is, we operate in a vertically differentiated product space. The analysis does not straightforwardly carry over to a horizontally differentiated space, since in that case issues of suitability for a particular customer arise. In fact, we need a single-crossing property to hold; and it is difficult to see how single-crossing could be made applicable to a horizontally differentiated product space.

### 1.4.3 Multiple Regulators

A recurrent theme of chapters 2–4 is the inefficiency caused by externalities that principals impose on each other through their non-cooperative behavior. The final chapter 7 takes up this issue in the context of the regulation of a single firm by two regulatory, or antitrust enforcement, agencies. The coordination problem between multiple regulatory authorities has been studied before (for instance, Martimort (1996b), Laffont and Martimort (1996)). In these models, each of several independent regulators seeks to influence the dimension of a firm's decision that is relevant to that regulator's constituency; for instance, an environmental regulatory agency may be concerned with a firm's emissions, while a price regulator may be concerned with the price the firm charges for its output. But none of the available models address the situation where one regulatory agency has greater power, or discretion, than another. This is the case, for instance, in European competition law enforcement. The principle of subsidiarity in European competition law enforcement implies that member states' competition authorities are subordinate to their European counterpart (the European Commission).[8]

We therefore seek to model a situation in which two regulatory agencies (price-)regulate a single firm, but one of the regulatory authorities (say, an international enforcement agency such as the European Commission) has greater discretion than the other (say, a member state's industry regulator). Generally, one should expect the objectives of the two regulatory agencies to differ: for instance, the industry regulator has a concern only for the citizens of its own country, while the international authority cares for the welfare of all citizens in the international federation. On the other hand, the subordinate industry regulator typically has greater investigatory powers, that is, can obtain better information at a lower cost than the international authority. Generally, we would therefore expect the international authority to make use of the national agency's information. But if the international authority were to overrule the national regulator on all occasions, the national authority would have no incentive to collect information. In equilibrium, we should therefore expect some, but not full, regulatory effort from both authorities. One somewhat counterintuitive implication this has for welfare is that the introduction of an international watchdog to "regulate the

[8] We review the structure of European competition law enforcement briefly in chapter 7.

regulators" may not necessarily be welfare improving.

From a modelling perspective, we address a hierarchical setting similar to that of Tirole (1986), but we introduce common agency concerns into the model. We do not know of a model that addresses a similar issue. From the perspective of applied theory, we also contribute to an understanding of the action space of an international competition authority, such as the European Commission. Although we focus on the price-regulating role of the international authority, we believe that we are taking an important step towards understanding the incentives for an international authority that is both charged with industry oversight and a mandate to "regulate the regulators."

# Chapter 2

# Common Agency under Symmetric Information

In this chapter, we begin our review of the theoretical literature on common agency. It is classical that the uncoordinated actions of individuals maximizing their private benefit leads to inefficiencies. In common agency models, where several principals non-cooperatively seek to control the decisions of a single common agent, we should expect similar inefficiencies if there does not exist a mechanism that internalizes the externality principals impose on each other through their non-cooperatively chosen actions. Later chapters will introduce an additional source of inefficiencies into the model, *viz.* those arising from non-cooperatively chosen ways of influencing the agent under asymmetric information. Here, we focus just on the question of inefficiency arising purely from the externalities principals create for each other in a world of full information.

Two papers provide the theoretical foundation for a burgeoning applied literature: Bernheim and Whinston (1986b) study common agency problems in a partial equilibrium framework; the paper by Dixit, Grossman, and Helpman (1997) generalizes the earlier results of Bernheim and Whinston considerably to a general equilibrium approach with non-transferable utility. Because of their importance for current applied work, we review both papers before considering some of the applications the models have found.

In both models, several principals make transfers to a single common agent in exchange for agent actions that impact on all principals' utility functions. Some obvious examples of such situations of common agency are the competition of pressure groups for policy favors from the policy maker; the competition (for instance, in tax schedules) of several governments for the choice of location of a multinational company; the influence manufacturers seek to exert on a common retailer; and so on. Two questions that naturally arise in this context are: (i) the characterization of equilibria in this game between principals, and (ii) whether some (of the potentially many) equilibria result in efficient allocations, and whether these equilibria seem in some sense focal or attractive.

Both Bernheim and Whinston (1986b) and Dixit, Grossman, and Helpman (1997) contain efficiency results: both papers show that for a certain attractive ("focal") refinement of Nash equilibria in the game between principals, an equilibrium exists that results in the efficient (first best) allocation. This refinement is a "truthfulness" requirement: It requires principals, when determining their transfer schedules to the agent, to reflect truthfully their preferences over the different allocations the agent could choose. Furthermore, truthfulness is an attractive requirement because Nash equilibria with this property are essentially the only equilibria that are collusion-proof in the sense that no group of individuals has an incentive to coordinate actions and deviate jointly from the (truthful) equilibrium strategies.

In terms of our (Coasean) organizing example, we might think of a situation where several ($I$) record producers compete for airtime allotted to them by the single D.J. The allocation of airtime to a single producer may be conceptualized as the fraction of total program length that is dedicated to playing that producer's record. The different decisions the agent can take are then elements of the $I$-dimensional unit simplex. Airtime, of course, is positively related to payoffs, so that each principal's payoff is increasing in its element of the decision vector and, if there is product market competition, decreasing in its competitors' allotted airtime. However, payoffs are also related to record quality: more airtime is of higher value to the producer who seeks to introduce a good record to the market, than to the producer who has a lower-quality record. The efficiency results hinted at above then imply that producers with higher quality records will obtain more airtime, and this is, of course, efficient.

The chapter is organized as follows: sections 2.1 and 2.2 review the common agency models of Bernheim and Whinston (1986b) and Dixit, Grossman, and Helpman (1997). Section 2.4 discusses some of the applications of the general model found in the literature.

## 2.1 Common Agency with Symmetric Information: Menu Auctions

Bernheim and Whinston (1986b) study a common agency model of symmetric information with transferable utility. Several principals simultaneously and non-cooperatively submit bids for the actions of a single common agent. There is an obvious analogy to auction theory, but note that the agent auctions off a vector of decisions (and not, as is usual in auction theory, a single indivisible object). Each principal therefore submits a menu of bids, conditional on the agent's decision vector. The common agency model of Bernheim and Whinston can therefore be studied as a "menu" auction. The particular auction Bernheim and Whinston analyze is a first-price menu auction, in which each principal pays the transfer payment she has submitted for the decision vector chosen by the agent. The agent is interested in maximizing her private benefit (the sum of transfer payments less her cost of implementing the chosen decision vector), while each principal seeks to maximize her net benefit (benefit from the chosen decision vector less transfer payment to the agent). Given this setup, Bernheim and Whinston study a complete characterization of the Nash equilibria of this two-stage game: First, principals design transfer schedules conditional on decisions. At the second stage, the agent selects a decision given transfer schedules.

### 2.1.1 The Model

Several principals $i = 1, ..., I$ simultaneously engage in a game of choosing conditional transfer payments to the single common agent, that is, transfer payments conditional on the decision taken by the agent. Denote by $\mathfrak{I}$ the set $\mathfrak{I} = \{i\}_{i=1}^{I}$ of all principals, and generically denote any subset of principals $J \subseteq \mathfrak{I}$, with complement $\bar{J}$ (and the complement of the singleton subset $J = \{i\}$ is denoted $-i$). Each principal $i$ chooses a transfer payment $t_i$ to maximize her (quasilinear) utility $n_i = v_i - t_i$.[1] Of course, utility depends on a decision (or action) $a$ taken by the agent, and transfers are made conditional on this decision, so that we write $v_i(a)$ and $t_i(a)$. Let $T_J(a) \equiv \sum_{i \in J} t_i(a)$. Generally, upper-case letters refer to sums of lower-case variables, so that the sum of gross payoffs to principals in set $J$ is

[1] Here, as indeed throughout the following chapters, we denote variables controlled by (or pertaining to) principal $i$ by a subscript $i$. Occasionally, we break with this convention when subscripts are used to denote partial derivatives, in which case the superscript $i$ is used to denote principal $i$'s variables.

$V_J(a) \equiv \sum_{i \in J} v_i(a)$.

While principals can make transfers to the agent, they cannot extract direct payments from the agent, so that for all $a \in A$ and all $i \in \mathfrak{I}$, $t_i(a) \geq 0$.[2]

The single agent chooses a decision (or action) vector $a \in A$, at a private cost of $c(a)$. She therefore seeks to

$$\max_{a \in A} u(a) - c(a) \equiv T_{\mathfrak{I}}(a) - c(a).$$

Denote the set of optimal agent actions (for given transfer schedules) $A^*(\{t_i\}_{i=1}^I) \equiv \arg\max_{a \in A}[T_{\mathfrak{I}}(a) - c(a)]$. Note that this choice need not be efficient: The set of efficient agent decisions (efficient for the agent and subset $J$ of principals) would of course be the joint surplus maximizing one: $A^J \equiv \arg\max_{a \in A}[V_J(a) - c(a)]$, and denote the overall efficient action set $A^* \equiv A^{\mathfrak{I}}$. The question of efficiency therefore reduces to a comparison of $A^*$ and $A^*(\{t_i\}_{i=1}^I)$, with those transfer schedules that are chosen in equilibrium. We therefore need to turn next to a characterization of equilibrium in this game.

### 2.1.2 Equilibrium Analysis

Nash equilibrium in this game is defined as it is usually by the condition that $(\{t_i^0\}_{i=1}^I, a^0)$ is a Nash equilibrium if no bidder wants to deviate from the equilibrium choices, that is if $v_i(a^0) - t_i^0(a^0) \geq v_i(a) - t_i(a)$ and $a^0 \in A^*(\{t_i^0\}_{i=1}^I)$.[3] Typically, however, the number of equilibria in this game is large. The way to pare down the number of equilibria in Bernheim and Whinston (1986b) is to focus on a subset of equilibria with a certain intuitive appeal: this is the subset of all "truthful" equilibria, that is, equilibria in which all players play truthful strategies. Bernheim and Whinston make the following definition:

**Definition 1** *A strategy $t_i$ is* truthful relative to $a^0$ *if for all feasible $t$ either*

$$v_i(a) - t_i(a) = v_i(a^0) - t_i(a^0)$$

[2] This is without loss of generality. In fact, Bernheim and Whinston (1986b) study a more general model in which the restriction is that $t_i(a) \geq k_i$. Furthermore, it is straightforward to prove that any such model is isomorphic to a model in which $k_i = 0$.

[3] And, of course, these strategies have to be feasible.

*or*

$$v_i(a) - t_i(a) < v_i(a^0) - t_i(a^0) \textit{ and } t_i(a) = 0$$

*holds.*

Put simply, a strategy is truthful if it reflects the relative payoffs (relative to the equilibrium gross payoff) to the principals of any possible (off-equilibrium) action choice of the agent. This is the first part of the definition. The only exception to this is when $t_i$ has reached its lower bound of 0, so that it cannot be lowered further. This is the second part of the definition. Of course the interpretation of a truthful strategy is just as the principal's compensating variation from different agent actions: the money transfer that leaves the principal just as well off as if the agent had chosen the reference action $a^0$. In effect, no principal is allowed to threaten with out-of-equilibrium offers that do not reflect the principal's true (relative) valuation of the out-of-equilibrium agent decisions.

The attractiveness of truthful strategies comes from two results: (i) every principal $i$ always has a truthful strategy amongst her set of best-response functions to any combination of her rivals' $(-i)$ strategies; and (ii) equilibria in which truthful strategies are played are coalition-proof. We focus first on the availability of truthful strategies and discuss coalition-proofness later.

Bernheim and Whinston (1986b) prove straightforwardly that each principal $i$ has amongst her best response set to others' strategies a truthful strategy. This property provides a certain focal character for truthful equilibria, since they are simple to calculate and truthful strategies are always available as a best response. To see why a truthful strategy always has to be available, consider a non-truthful equilibrium strategy. Define a new truthful strategy that implies the same equilibrium payment to the principal (but may conceivably differ off-equilibrium). If the agent still chooses the same equilibrium action, therefore, the truthful strategy gives the principal the same payoff. If switching to a truthful strategy implies that the agent chooses a different action, this gives the principal the same payoff since the new net payoff (by definition of truthfulness) equals the net payoff if the original equilibrium action had been chosen.

This equilibrium refinement allows a reduction in the set of equilibria in this game. Bernheim and Whinston show in particular that in all truthful equilibria, an action $a^0 \in A^*$ is chosen. That is, truthful equilibria always result in efficient agent actions. Furthermore, Bernheim and Whinston also have the following efficiency result about net payoffs to the principals: the principals' (net) payoffs are efficient in the sense that there does not exist another distribution of payoffs across principals that leaves no principal worse off, and which has the property that no subgroup of principals obtains a payoff that exceeds the total surplus which that subgroup adds to the total surplus that could have been created without that subgroup. That is, in a truthful equilibrium, the distribution of net payoffs to all bidders is a Pareto optimal one, among all those distributions that give no subset of principals a higher joint payoff than their contribution to joint surplus. In brief, all net payoffs are in the set (for some $a \in A^*$) $E(a) \equiv \{n \in R^I | n \in \Pi(a)$ and there does not exist $n' \in \Pi_\Gamma(a)$, with $n' \geq n\}$, where $\Pi(a) \equiv \{n \in \Re^I|$ for all $J \subseteq \mathfrak{I}$, $N_J \leq [V_{\mathfrak{I}}(a) - c(a)] - [V_{\bar{J}}(a^{\bar{J}}) - c(a^{\bar{J}})]\}$. Furthermore, any member of the set $E(A^*)$ can be supported by a truthful equilibrium. So not only will efficient actions be chosen in equilibrium, but the resulting payoffs to principals are also Pareto efficient.

### Coalition-Proofness

Finally, Bernheim and Whinston (1986b) argue for their equilibrium refinement ("truthfulness") from a standpoint of stability. The kind of stability Bernheim and Whinston have in mind is stability when coalition-formation (that is, communication and binding agreements amongst subsets of principals) is possible. This stability requirement ("coalition-proofness") is of course just another equilibrium refinement, albeit an attractive one in this context. Specifically, the refinement requires that any equilibrium with this property yield a Pareto optimal distribution of net payoffs to all principals, among those Nash equilibria in which no proper subset of players has a jointly optimal deviation (that is would want to collusively deviate from the equilibrium strategy). Bernheim and Whinston show that all truthful equilibria have this stability property, and that the agent's action choice (and the principals' payoffs) are isomorphic in truthful equilibria and in coalition-proof equilibria. That is, both equilibrium refinements result in payoffs in the set $E(A^*)$.

## 2.2 Common Agency with Symmetric Information: Non-Transferable Utility

While the Bernheim and Whinston model has proven influential in applications to situations of political influence such as tax policy or strategic trade policy, the model does not speak on distributional issues. Since in the Bernheim and Whinston (1986b) framework utility is directly transferable (through straightforward monetary transfers), distributional concerns cannot be addressed in this model. Yet, these distributional concerns are at the heart of the questions that the model has been applied to. It is therefore important to have a generalization of the model that permits statements about the distribution of wealth. Dixit, Grossman, and Helpman (1997) provide an important generalization of the Bernheim and Whinston (1986b) paper, and we therefore here briefly review their model.[4]

Just as in Bernheim and Whinston (1986b), Dixit, Grossman, and Helpman (1997) model the game of mechanism design between $I$ principals who non-cooperatively seek to control the actions of a single agent under conditions of perfect and complete information. The timing of the game is the same as before: the principals design payment schedules for an agent who, given payment schedules, takes a decision that influences the payoffs of all principals. Again, this game of mechanism design has a potentially large number of Nash equilibria. The refinement that is used to pare down the multiplicity of equilibria is, as in Bernheim and Whinston (1986b), the idea of truthfulness. In this framework also, a parallel efficiency result obtains: truthful equilibria are Pareto efficient in the sense that any other strategy choices by any of the players (principals or agent) would result in lower payoffs for at least some of the players.

### 2.2.1 The Model

As before, denote by $\mathfrak{I}$ the set of principals $i = 1, ..., I$. Since utility is assumed non-transferable, each principal $i$ has preferences defined over the agent's action vector $a$ and her payments to the agent $t_i$, represented by $v_i(a, t_i)$. At the first stage of the game, principal $i$ chooses a payment function $t_i(a) \in \mathfrak{T}_i$, where $\mathfrak{T}_i$ reflects institutional constraints

[4]The published paper (Dixit, Grossman, and Helpman (1997)) unfortunately does not contain the proofs of the propositions. The reader is therefore referred to the working paper (Dixit, Grossman, and Helpman (1996)).

on the feasibility of payment functions (for instance, when principals can make transfers to the agent, but not extract money from the agent, i.e. $t_i \geq 0$, $\mathfrak{T}_i$ would be the set of all nonnegative-valued functions, etc.).

The agent's utility function over her action (vector) and the principals' payments is $u(a,t)$. Given transfer schedules by the principals, the agent seeks to choose an action $a \in A$ to maximize $u(a,t)$.

### 2.2.2 Equilibrium Analysis

Dixit, Grossman, and Helpman (1997) study Nash equilibrium in the game amongst principals, given that the agent responds to the given transfer schedules by choosing her action in a utility maximizing way. That is, the set of transfer schedules $t^0(\cdot) = \{t_i^0(\cdot)\}_{i \in \mathfrak{I}}$ and the agent action $a^0$ constitute a Nash equilibrium $(t^0(\cdot), a^0)$ if, and only if, the transfers are feasible ($t_i^0(\cdot) \in \mathfrak{T}_i$ for all $i \in \mathfrak{I}$), the agent maximizes $a^0 \in \arg\max_{a \in A} u[a, t^0(a)]$, and every principal $i \in \mathfrak{I}$ chooses her transfer schedule such that $v_i[a^0, t_i^0(a^0)] \geq v_i[a_i, t_i(a_i)]$ where $a_i$ is defined as $a_i \in \arg\max_{a \in A} u[a, (\{t_j^0(a)\}_{j \neq i}, t_i(a))]$. (This last condition says that every principal chooses her best response to the other principals' strategies, given that if she alone were to change her strategy, the agent would respond by choosing a different action $a_i$, and that no such deviation should give the principal a higher payoff.)

Dixit, Grossman, and Helpman (1997) prove the following equilibrium characterization, which replaces the best-response requirement of the equilibrium definition with a condition that allows interpretation of the equilibrium in terms of the relationship between any one principal and the agent. The intuition for this condition is the following: each principal's best response to the other principals' transfer schedules is to induce the agent to choose the action (and to choose a transfer) that maximizes that principal's utility, given that she cannot induce an action choice that does not give the agent at least her outside option (that is, at least as much as the agent would get if that principal contributed nothing). In particular, this condition implies that, in equilibrium, the agent obtains a payoff that is just as large as if one principal contributed nothing, with all other principals maintaining their transfer schedules (and the agent re-optimizes accordingly). The following proposition summarizes:

**Proposition 2 (Dixit, Grossman, and Helpman)** *$t^0(\cdot) = \{t_i^0(\cdot)\}_{i\in\mathfrak{I}}$ and $a^0$ are an equilibrium if and only if: (i) $t_i^0(\cdot) \in \mathfrak{T}_i$ for all $i \in \mathfrak{I}$; (ii) $a^0 \in \arg\max_{a\in A} u[a, t^0(a)]$; and (iii) for every $i \in \mathfrak{I}$:*

$$[a^0, t_i^0(a^0)] = \arg\max_{(a,t)} v_i(a,t),$$

*s.t. $a \in A$, $t = t_i(a)$ for some $t_i(\cdot) \in \mathfrak{T}_i$, and*

$$u[a, (\{t_j^0(a)\}_{j\neq i}, t)] \geq \max_{a'\in A} u[a', (\{t_j^0(a')\}_{j\neq i}, 0)].$$

Of course this implies that in equilibrium, the agent obtains just $u[a^0, t^0(a^0)] = \max_{a\in A} u[a, (\{t_j^0(a)\}_{j\neq i}, 0)]$ because it is in none of the principals' interest to give the agent more than necessary.

As before, the equilibrium is plagued by a multiplicity of possible Nash equilibria. Furthermore, condition (iii) in proposition 2 involves maximization over all payment functions, which is computationally intense. Dixit, Grossman, and Helpman (1997) therefore argue for truthfulness as an appropriate refinement on the grounds that truthful equilibria are focal in the sense that every principal's best response set to other principals' strategies always contains a truthful strategy. The definition of truthfulness is a precise analogue to that used in Bernheim and Whinston (1986b), with an obvious change in notation that simplifies interpretation when utility is non-separable in transfers. Specifically, they define a truthful transfer schedule to be one which reflects precisely (in money terms), the changes in the principal's utility from different agent actions. Again, the interpretation is just that introduced earlier: a truthful transfer is just the compensating variation for different agent actions, relative to the principal's utility level in equilibrium. Precisely, the definition is:

**Definition 3** *A transfer schedule $t_i^T(a, v_i^*)$ is* truthful *relative to utility level $v_i^*$ if, for all $a \in A$*

$$t_i^T(a, v_i^*) \equiv \min[\bar{t}_i(a), \max[0, \varphi_i(a, v_i^*)]]$$

*where $\varphi_i$ is implicitly defined by*

$$v_i[a, \varphi_i(a, v_i^*)] = v_i^* \text{ for all } a \in A,$$

*and*

$$\bar{t}_i(a) = \sup\{t_i(a) | t_i \in \mathfrak{T}_i\} \text{ for all } a \in A.$$

(The definition also ensures that truthful schedules are feasible.) Truthful equilibrium is similarly defined as a Nash equilibrium in which truthful strategies are played.

In analogous fashion to the result of Bernheim and Whinston, existence of a truthful strategy in every principal's best response set to every combination of other principals' equilibrium (not necessarily truthful) strategies can be proven. This allows a focus on truthful strategies as in some sense "focal." Focusing on truthful equilibria reduces the number of equilibria in this game and simplifies calculation of the equilibrium payoffs. Consider again proposition 2. It states that in equilibrium, the agent's utility is just what she could obtain if one principal contributed nothing, while all other principals maintain their equilibrium transfer schedules. Now, we can replace the equilibrium transfer schedules in that proposition by the utility numbers achievable in a truthful equilibrium (recall that $t_i^T(a, v_i^0)$ only depends on the equilibrium utility level $v_i^0$).

Truthfulness in this framework without transferable utility also has the desirable property that the resulting allocation is efficient in the sense that no other feasible action and transfer schedule choice exists that increases everyone's (principals' and agent's) payoffs. In brief, Dixit, Grossman, and Helpman (1997) prove the following:

**Proposition 4 (Dixit, Grossman, and Helpman)** *Let a policy vector $a^0$ and a vector of payment functions $t^0(\cdot)$ that are truthful with respect to the utility levels $v_i^0 = v_i(a^0, t_i^0(a^0))$ constitute a truthful equilibrium. Then there do not exist an action $\tilde{a}$ and a payment vector $\tilde{t}$ such that (i) they are feasible:*

$$\tilde{a} \in A; \text{ and } 0 \leq \tilde{t}_i \leq \bar{t}_i(\tilde{a}) \text{ for all } i \in \mathfrak{I};$$

*and such that (ii) they constitute a Pareto improvement:*

$$u(\tilde{a}, \tilde{t}) \geq u[a^0, t^0(a^0)],$$

$$v_i(\tilde{a}, \tilde{t}_i) \geq v_i[a^0, t_i^0(a^0)] \text{ for all } i \in \mathfrak{I},$$

*with at least one strict inequality.*

In some sense, of course, the efficiency results of both Bernheim and Whinston (1986b) and Dixit, Grossman, and Helpman (1997) are obvious, given truthfulness: when principals submit truthful transfer schedules, each principal contributes to the agent action (which has the qualities of a public good, since it has implications for each principal's payoff) just her marginal willingness to pay. This is just the familiar condition for optimality in this context: truthful payments equate each principal's marginal utility with the marginal payment to the agent. Since in equilibrium, each principal's payment is, at the margin, just what induces the equilibrium agent action (recall the implication of proposition 2), the agent's maximization problem therefore equates the marginal cost (to her) and the marginal payment of each principal. Of course, the efficiency results have strong implications: for instance, in applications to political processes that determine economic policies, lobbying by special interest groups results in an efficient outcome (although the distributional implications may be very different).

## 2.3 Interpretation

We began with the observation that the uncoordinated actions of utility maximizing agents generally results in inefficient allocations, even under symmetric information. The remarkable feature of the two models we have reviewed above is that a suitable equilibrium refinement (truthfulness) internalizes the externalities that several principals impose on each other through uncoordinated mechanism design (or "bidding") for the single agent's action.

When we consider common agency models under asymmetric information in the following two chapters, efficiency will prove more elusive. The reason for this is simple: while the uncoordinated actions of several principals under symmetric information can easily by internalized through the truthfulness requirement, the informational incompleteness assumption introduces inefficiencies of its own. Importantly, even without the non-cooperative behavior of principals, inefficiencies result from the informational asymmetry. When principals design mechanisms in competition with each other, as well as under conditions of asymmetric information, we should generally expect different inefficiencies to arise from the externalities principals impose on each other. However, it is not clear how these inefficiencies can be internalized through a truthfulness refinement: it is not clear what "truthfulness" implies in an asymmetric information setting in which principals' transfer schedules are designed so as to elicit the agent's private information.

However, the efficiency results of Bernheim and Whinston (1986b) and Dixit, Grossman, and Helpman (1997), and the relative simplicity of the models, have proved attractive for researchers in a very diverse set of applications. We next turn to a brief review of some of the applications of the theory.

## 2.4 Applications

The literature applying the framework developed in Bernheim and Whinston (1986b) and Dixit, Grossman, and Helpman (1997) is large. In the following sections we review some of the applications the theory has found, but do not discuss all in detail. The section is organized by area of application.

### 2.4.1 Common Sales Agents

A natural application of the common agency framework is to the problem of choice of retailing structure, when firms delegate their marketing decisions.

Although a precursor to the very general models of Bernheim and Whinston (1986b) and Dixit, Grossman, and Helpman (1997), an interesting application of the common agency methodology to common retailers is the paper by Bernheim and Whinston (1985).

Bernheim and Whinston (1985) model the choice of retailing structure in a model without informational asymmetry. Each principal $i = 1, 2$ sets a sale price $p_i$ and delegates the marketing decision $m_i$ to a risk-neutral agent who incurs cost $c_i(m_i)$ and obtains reward $I_i(m_i, x_i)$. Demand is random, $x_i = D_i(p, m)$, and $i$'s production cost is $\Gamma_i(x_i) = \gamma_i x_i + F_i$.

Conditioning of contracts is only possible on a principal's own variables (say, because of competition law regulations), and on the retailing structure that is chosen (that is on whether the agent chooses also to sell the rival principal's product or not). Importantly, principals choose whether or not to employ a common sales agent.

The game of retailing structure Bernheim and Whinston analyze is the following: firms offer contracts $(p_i, I_i)$ for each configuration of the market (principal-agent groupings), some of which will be accepted by the agents, and finally each firm chooses one of the agents willing to accept its offer. The method Bernheim and Whinston employ to solve this game is first to analyze a simplified model of intrinsic common agency (that is, a model in which all principals contract with the same agent, and this agent cannot refuse offers), and then to show that the solution of this model also represents an equilibrium in the larger model in which retailing structure is a choice for the principals.

In this simplified, intrinsic common agency game (in which all principals have to contract with the single agent), Bernheim and Whinston (1985) show that there exists an equilibrium in which principals choose collusive prices (and induce collusive levels of marketing intensity), and, despite observability of the agent's action, offer reward schemes based only on outcome ("commission") and a constant "sell out" or "franchise" fee, and that this choice is endogenous to the model. This characterization is intuitive: Since principals may not condition contracts on each other's variables, conditioning the incentive scheme on sales allows principal $i$ to condition indirectly on the marketing decision $m_{-i}$ of the agent in the rival principal's incentive scheme. (Note also that agents are assumed risk-neutral.)[5]

The central result of Bernheim and Whinston's paper is that the principal's problem in this simplified common agency game is nested in the principal's problem in the general game in which retailing structure is a choice variable for principals. That is, that there exists a separating equilibrium in the general game that has the fully collusive outcome with incentive schemes of the sort described above (franchise fee and commission-based reward).

In their model, the common agent allows a collusive outcome. This is intuitive: Since there exists an equilibrium in which principals essentially sell the firm to the agent (through

[5] This is a standard result for risk-neutral agents in moral-hazard problems with hidden action; cf. chapter 3.

the franchise fee-plus-sales commission contract), in this equilibrium each principal now only chooses its price. Price extracts surplus from the agent. As is usual, one principal chooses its price, taking the rival principal's choice as fixed, that is, we describe the principals' choice of a point from the set of Pareto efficient outcomes. Principals therefore set prices so as to maximize the total value of the surplus they can extract, that is they choose the co-operative (or "collusive") price.

Zhang (1993), in a model of common sales agency, shows that sales agents fulfil not only a collusive role (as in Bernheim and Whinston (1985)) but that common sales agency may be chosen because of its precommitment value to the principals. In Zhang's model, products of two principals are horizontally differentiated, so that for each customer, one product always provides a better match than the other. Buyers, however, are badly informed about the suitability of each product for them. Retailers may bridge this informational gap. However, an exclusive agent has no incentive to turn a customer away, because she will lose the sale. A common agent lacks this incentive, and can advise in a more unbiased way (depending on the compensation schemes she is given by the two competing principals). In general, one might therefore expect some marginal customers (who do not purchase the good under exclusive sales agency) to purchase under common sales agency. Since sales agency therefore increases the size of the pie to be divided by the principals, it should always be chosen. What Zhang also demonstrates is (and this is intuitive from the foregoing discussion) that there exists a symmetric common agency equilibrium, in which principals pay identical compensation schemes, and in which sales agents give entirely unbiased advice.

Zhang's model, while providing an answer to the question of the incentives that common agency settings create for the quality of advice a common retailer provides, does not apply to a large class of cases in which repeat purchases are important. Essentially, when repeat purchases matter, a different process needs to be modelled, *viz.* that whereby principals choose compensation schedules for their retailers that attract first-time buyers (who will then repeat purchase in the future). Many problems in which common sales agency is important turn out to be of that kind: common sales agents for insurance contracts; common retailers for savings products; travel agents; etc. A model which focuses on this aspect of common sales agency is presented in chapter 6.

### 2.4.2 Strategic Trade Policy

Grossman and Helpman (1994) use the Bernheim and Whinston (1986b) model to study the trade policy tools chosen in a small, competitive, open economy that produces $I$ different outputs, each from one of $I$ sector-specific inputs. Lobby groups representing the owners of a sector-specific input can make monetary contributions to the government, conditional on the policy vector chosen by the government. The incumbent government is interested in maximizing its re-election prospects and therefore maximizes a weighted sum of total monetary contributions from lobby groups and social welfare. Interestingly, the government's choice here (unlike in the paper by Bernheim and Whinston where choices are discrete) is a continuous choice of import and export taxes or subsidies. In effect, the government chooses a domestic price vector: domestic prices above world prices imply import taxes and export subsidies, and domestic prices below world prices imply import subsidies and export taxes. Clearly, the domestic price for a good influences the payoff of the lobby group that owns the input used in the production of that good.

Grossman and Helpman refine Nash equilibrium in contribution schedules using only the concept of local truthfulness (that is, contribution schedules need only reflect the lobby group's marginal rate of substitution near the equilibrium point, not globally). This refinement is sufficient to derive the policies used in equilibrium: in general, the taxes (or subsidies) used in equilibrium will be greater the lower the import demand or export supply elasticities in that sector. This is intuitive: if these elasticities are large, distortionary taxes cause greater welfare loss. Secondly, greater elasticities will cause greater welfare losses from taxation, so that lobby groups in other sectors (who share in this loss) would wish to increase their contributions to avoid protection in the sector with large supply or demand elasticities. Finally, Grossman and Helpman (1994) provide examples of the power of lobby groups: when there is only one lobby, it captures all available gains from the political relationship with the government; when all individuals are represented by some lobby, competition for favorable policy decisions is so intense that the government captures all of the surplus from the political relationships.

Grossman and Helpman use a similar framework in their paper on the interaction of national political leaders in the international arena, when national interest groups can make conditional monetary contributions (Grossman and Helpman (1995b)), and in their paper on the negotiation of trade agreements between countries when governments are influenced by lobby groups' contributions (Grossman and Helpman (1995a)). Further applications of the Grossman and Helpman (1994) model to the study of lobbying over environmental policy are the papers by Aidt (1998) and Fredriksson (1999). Both apply the framework developed by Grossman and Helpman straightfowardly, and obtain the expected conclusions on the political influence of pressure groups.

In a brief application of the Bernheim and Whinston (1986b) model, Baron (1997) studies the competition between the US and Japan over favorable trade policy for its producers of photographic film, Kodak and Fujifilm in the Japanese market. Both Kodak and Fujifilm can lobby both governments for favorable trade decisions (sanctions), and in this sense the model is one of competing common agency: the common agents (that is, the governments) compete against each other. The equilibrium characterization in Bernheim and Whinston (1986b) is applied straightforwardly to this case. The treatment of the common agency aspect of competition between Kodak and Fujifilm is brief: this "nonmarket" strategy of lobbying for favorable policy outcomes is only one aspect of their competition, and the two competitors also engage in direct market interaction. Much of the interest of the paper lies in its (informal) discussion of the interaction between market and nonmarket strategies. Largely, though, these concerns are orthogonal to those discussed in this chapter.

### 2.4.3 Tax Competition

In a common agency model with perfectly and symmetrically informed principals, Haaparanta (1996) develops a model of tax competition. Two national governments compete for the share of investment allocated to their respective countries by a multinational corporation. Governments compete in subsidy schedules conditional on the investment share they receive. Each country has fixed market size and a fixed wage rate. Governments maximize labor income net of subsidies, while the multinational maximizes profits (defined in the conventional way). Using the equilibrium characterization in Bernheim and Whinston (1986b), Haaparanta shows that in a (truthful) Nash equilibrium in subsidy schedules, increasing the wage rate will result in lower foreign direct investment if the impact of investment on the marginal productivity of labor is small. The intuition for this result is simple: if investment has little effect on marginal productivity, investment will not increase the demand for labor by more than the subsidy needed to attract that investment. In equilibrium, subsidies (and investment) are likely to be low.

The amount of investment in the equilibrium in subsidy schedules depends on the firm's production technology. For instance, a Leontief technology results in higher investment in high-wage countries: equilibrium subsidies remove the effect of wage differentials on investment; a Cobb-Douglas technology results in the same allocation of investment with or without subsidies. The intuition is the same as before: Leontief technology implies large changes in the marginal productivity of labor for given changes of investment, so that subsidies are effective in increasing the profitability of investment. With Cobb-Douglas technology, the impact on the marginal productivity of labor decreases with the wage rate, so that subsidies are less effective for high wage countries.

The level of equilibrium subsidies again is ambiguous: starting from a position of wage equality, increasing one country's wage may result in either higher or lower equilibrium subsidies being paid. This is certainly the case when both countries have very similar wage levels: the higher wage country pays higher subsidies, regardless of whether this increases or reduces its share of investment.

### 2.4.4 Special Interest Politics

In a model of special interest politics, Persson (1998) models the provision of public goods, where lobby groups (each group is defined through its consumption of one particular public good) can lobby for public good provision. For instance, the public goods might be local public goods, and the lobby groups regional pressure groups that benefit from the local public good. Persson derives a truthful equilibrium in contribution schedules from lobbies and shows that typically public goods are misallocated, with lobby groups obtaining more than the social optimum and unorganized non-lobbyers obtaining less. Also, the fewer lobby groups there are the worse the misallocation to that group: a larger number of lobby groups would internalize the allocational inefficiencies. This result precisely mirrors those of Grossman and Helpman (1994), above. Persson then embeds these results in a larger model of elections for the legislature in a presidential system. However, these concerns of the paper are largely orthogonal to our own concerns here.

### 2.4.5 Multiple Regulators

Rizzo and Sindelar (1996) provide an interesting application of a common agency model to the regulation of healthcare markets. In particular, they study the co-ordination failures that arise when an industry is regulated by several principals with different objectives. In a model without asymmetric information between regulated firms (in their model, physicians) and industry regulators (the US Health Care Financing Administration, HCFA, and the US Agency for Health Care Policy and Research, AHCPR), Rizzo and Sindelar model the impact on quantity and quality of physician services. In a very tightly specified model, they show that the AHCPR's concern for quantity and cost of physician services, and the HCFA's concern for cost control and quality of service (e.g. accessibility, patient satisfaction) result in quantity and quality above the social optimum, and in a price (and therefore, other attributes of physician services such as location of practice, condition of facilities, etc.) below the social optimum.

# Chapter 3

# Moral Hazard

This chapter considers hidden action models of moral hazard, that is, models in which a principal delegates decisions to one or more agents but cannot observe the agents' actions. In fact, throughout this chapter, we will focus on the case where a principal seeks to control a single agent's actions; the multi-agent case raises a different set of issues which are largely orthogonal to our concerns.[1] Actions result in outcomes (over which the principal's preferences are defined) according to some stochastic technology with positive variance. (If the variance of the technology were zero, the principal could infer the agents' actions precisely, and there would be no incentive problem, at least not in the single agent case.) Taking the "right" action (the action that is most likely to generate the outcome most preferred by the principal) is costly for the agent: in general, the agent would like to take decisions less preferred by the principal. Although the principal cannot observe actions directly (for instance because observation is prohibitively costly), the principal can provide incentives based on the observable outcomes of the agent's actions. Observable outcomes are partly the result of the agent's actions, and partly the result of "luck;" and we should therefore expect the principal to reward the agent for good outcomes, and punish for bad outcomes. But outcome-based rewards impose risk on the agent; and for risk-averse agents, incentives for effort result in sub-optimal levels of risk-sharing. In general, monetary transfers to the agent have to be increased to satisfy the agent's participation decision. The design problem

[1]For instance, observing only the joint output of a team of agents raises issues of free riding. Conversely, observing output of individual agents in teams (subject to correlated output shocks) may improve the principal's information about individual performance (Holmstrom (1982)).

is to balance the incentives for the "right" action against the cost imposed on the agent from risk-bearing.

In fact, most of the literature has studied a more restrictive case in which the agent's action is one-dimensional. We follow this convention and will therefore conveniently interpret action as "effort." High effort is more likely to result in good outcomes for the principal (for instance, profit), in the sense of stochastic dominance. In this case also, we should therefore expect the principal to reward the agent for high realizations of profit, and punish for low profit realizations. Again, the optimal incentive scheme balances increased incentives for high effort against the (risk-) cost imposed on the agent.

But the intuition to reward for higher realizations of the observable outcome and to punish for lower realizations, only holds up to a point. In general the principal might well wish to design a non-monotonic incentive schedule. Assumptions that are sufficient for monotonicity are known, but other desirable properties such as linearity of the optimal incentive scheme cannot in general be guaranteed.

The divergence between the simplicity of observed incentive schemes (for instance, the linearity of piece rates; or retailer's per-unit sales commissions) and the complex incentive schemes predicted by theory have prompted a search for the cause of this disparity. One reason for the relatively complicated incentive schedule that results from theory is the divergence between the choice sets of principal and agent: while the agent essentially is restricted to choosing a point on a path through the probability simplex (of distributions over outcomes), the principal maximizes over all real-valued functions defined over the set of outcomes.[2] Enlarging the agent's choice set may therefore result in simpler optimal incentive schemes, and possibly linearity. To build intuition, we begin our discussion of common agency under conditions of moral hazard in this linear incentive contracts framework, and then consider the general non-linear contracts case.

A common theme of common agency models under conditions of asymmetric information

[2]This intuition comes from what has come to be known as "Mirrlees's unpleasant theorem:" that for outcome distributions over the entire real line (e.g. the normal distribution with the agent's effort as its mean) and agent utilities that are unbounded from below, the first-best solution can be approximated arbitrarily closely through an incentive scheme that punishes infinitely for very low outcomes. For instance, normal distributions are completely informative in the tails: very low outcomes result from low effort with probability one. In order to discourage low effort, infinite punishments (that happen with sufficiently low probability so that the agent still wishes to participate in the scheme) deter the agent from low effort choice.

is that in general, non-cooperative contracting between principals creates externalities in incentive provision. We will encounter this inefficiency again in the next chapter, for models of asymmetric information. Here, the inefficiency arises because of the non-cooperative nature of incentive design: If principal 1 considers lowering her incentive payment to the agent, and for the agent, the unobservable activities are cost substitutes (lowering the action undertaken on behalf of principal 1 reduces the marginal cost of lowering the action undertaken on behalf of principal 2), then the agent will respond by lowering her action on behalf of principal 2 also. In equilibrium, both principals will foresee this and provide incentives that are more high-powered than in the second-best case. Similarly, if activities are cost complements, we should expect incentive schemes to be too low-powered relative to the second-best optimum. In general, therefore, common agency will lead to inefficiencies relative to the second-best optimum: the agent's performance will either be inefficiently low or inefficiently high. Both the general models that we discuss in this chapter have this feature. The other question we ask is what incentive schemes will be chosen in equilibrium. A result of great generality is that the aggregate incentive scheme that is used in equilibrium will be the cost-minimizing one.

Briefly return to our motivating example. In terms of our organizing example, we might envision a situation as follows. A D.J. does more than just play records; for instance, she will comment on each record (favorably or unfavorably), or place records in slots on her program that attract a larger audience (for instance, the first record after the hourly newscast). That is, she can spend effort that increases the expected sales of a record. Not all of this is easily observable for record producers; indeed, some of this may be information that is hard to verify and cannot be written into (legally enforceable) contracts. Information that is more easily available is the D.J.'s playlist, that is, count data on how often the D.J. has played any given record. On the other hand, information about the frequency of play of any given record is almost certain to be more informative about the D.J.'s level of effort than record sales figures (sales figures are aggregates of the effort levels of several D.J.s; they are the result of other means of advertising; etc.). Record producers may therefore wish to base incentive contracts on this count data (number of times a record is played).

The general conclusion from the models reviewed in this chapter should therefore be that, in general, the incentives given to the D.J. are inefficient, in the sense that the D.J.'s actions will be inefficient. However, as the general models predicts, the aggregate incentive scheme given to the D.J. will be the cost-minimizing one: although it may not result in the second-best optimal action, the (aggregate) incentive scheme is the least cost way of obtaining that agent action.

## 3.1 Outline

The chapter begins with a review of the theory of moral hazard with hidden action in the single-principal single-agent case. Section 3.2 presents the theory of the single-principal single-agent case. In an essentially static model of considerable generality, it turns out that the incentive scheme is difficult to characterize. The section therefore proceeds to analyze an extension to a multi-period model, in which incentives are given for performance over time. The resulting incentive scheme can be written, in a reduced form, as a linear scheme. This simplifies the analysis considerably. Section 3.3 studies the common agency case, in which several principals non-cooperatively provide incentives for the agent. The section begins by building intuition using the linear transfers case and then studies the problem with more general (non-linear) incentive schemes. Finally, we review the few direct applications of the theory that can be found in the literature.

## 3.2 The Single-Principal Single-Agent Moral Hazard Model

### 3.2.1 A General Model

In a general formulation of the problem, the principal's problem is to design a contract or schedule of payments (or sharing rule) to the agent contingent on observable and verifiable outcomes of the agent's unobservable action.[3] Since the principal knows the agent (knows her utility function), she effectively implements the desired action through appropriate contract design. The principal's problem is thus to choose actions and payment schedule so as to maximize her objective function subject to the constraints that (i) the agent obtain an expected utility not less than her reservation utility level (assumed exoge-

[3]The classic references are Mirrlees (1974), Mirrlees (1976), Mirrlees (1999). For a survey treatment, cf. Hart and Holmstrom (1987).

nously determined)—i.e. that her *individual rationality* (IR) (or participation constraint) be satisfied—and that (ii) given the contract, the agent in fact wishes to take the desired action—i.e. that the desired action be *incentive compatible* (IC). A sharing rule that satisfies constraints (IC) and (IR) for some action is said to implement that action. The principal's problem is thus to choose the optimal action from the set of actions that are implementable by the optimal sharing rule.

To be specific, let the agent choose an action $a \in A$ (for simplicity, it is common to make the further assumption that $A \subset \Re$). Actions are linked to observable outcomes via the stochastic technology $x = x(a;\theta)$, and to the principal's payoffs via the technology $\pi = \pi(a;\theta)$; both are functions of action and state of nature $\theta \in \Theta \subset \Re$, where $\theta$ is distributed according to the distribution function $G(\cdot)$ on $\Theta$. The principal designs a contract (or sharing rule, or transfer payment schedule) $t(x)$ of payments contingent on observable outcomes.[4] The principal's utility over wealth $m$ is $v(m)$, with $v'(\cdot) > 0$ and $v''(\cdot) \leq 0$. The agent's utility is commonly assumed to be separable in wealth and action, so that her utility function is $u(m) - c(a)$, with $u'(\cdot) > 0$, $u''(\cdot) < 0$,[5] and $c'(\cdot) > 0$. This is the *state-space formulation* of the problem.[6] The principal's problem is to

$$\max_{a,t(\cdot)} \int v(\pi(a;\theta) - t(x(a;\theta)))dG(\theta)$$

s.t.

$$\int [u(t(x(a;\theta))) - c(a)]\, dG(\theta) \geq \underline{u}$$

$$a \in \arg\max_{\hat{a}\in A} \int [u(t(x(\hat{a};\theta))) - c(\hat{a})]\, dG(\theta)$$

It has been more common, however, to use the *parameterized distribution function for-*

[4]In this literature, the sharing rule is customarily referred to as $s(\cdot)$. We here use $t(\cdot)$ for purposes of continuity with the following chapters.

[5]If the agent is risk neutral, the moral hazard problem can be avoided ( Harris and Raviv (1979, Proposition 3)). There exists a contract $t(x) = x - \alpha$ (where $\alpha$ is chosen so as to just satisfy the agent's individual rationality constraint), that provides the agent with first-best incentives. Note that this contract is a "selling out" contract: the agent receives the payoff to her action; and her payment to the principal does not depend on her action.

[6]cf. Wilson (1969), Spence and Zeckhauser (1971), Ross (1973)

*mulation* pioneered by Mirrlees (1974), which views the agent as controlling, through her choice of action, a distribution $F(x, \pi; a)$, with associated density $f(x, \pi; a)$, over observable and verifiable outcomes $x$ and monetary payoffs $\pi$ to the principal. The principal's problem is thus to

$$\max_{a, t(\cdot)} \int v(\pi - t(x)) f(x, \pi; a) dx \tag{3.1}$$

s.t.

$$\int [u(t(x)) - c(a)] f(x, \pi; a) dx \geq \underline{u} \tag{3.2}$$

$$a \in \arg\max_{\hat{a} \in A} \int [u(t(x)) - c(\hat{a})] f(x, \pi; \hat{a}) dx \tag{3.3}$$

The parameterized distribution function approach may appear too restrictive. The *general distribution function formulation* gives the agent the much richer strategy space of choosing directly a probability distribution (or density) over outcomes. It does not, therefore, restrict the agent to the linearity imposed by the parameterized distribution function approach. This approach, though conceptually rich, turns out to be analytically difficult. We therefore choose to discuss the problem in its parameterized distribution function formulation.

The program (3.1)–(3.3) is easily solvable for the discrete case where $A \equiv \{L, H\}$, $\pi \equiv x$, and $v''(\cdot) = 0$, the interpretation being that the agent chooses one of two actions (high or low effort), the observable outcome being the profit accruing to the (risk-neutral) principal. Given this simplification, assuming that the principal wishes to implement $a = H$,[7] and (since $v''(\cdot) = 0$ we can apply any positive affine transformation to the principal's utility) making an adequate choice of units for the principal's utility, the program (3.1) to (3.3) becomes:

$$\max_{t(\cdot)} \int (x - t(x)) f(x; H) dx$$

[7] If the principal wishes to implement $a = L$, she simply chooses a constant $t(x) = \alpha$ that just satisfies the agent's individual rationality constraint.

s.t.

$$\int [u(t(x)) - c(H)]\, f(x;H)dx \geq \underline{u}$$

$$\int [u(t(x)) - c(H)]\, f(x;H)dx \geq \int [u(t(x)) - c(L)]\, f(x;L)dx$$

Letting $\lambda$ and $\mu$ be the Lagrangian multipliers for constraints (3.2) and (3.3), respectively, the optimal transfer schedule satisfies[8]

$$\frac{1}{u'(t(x))} = \lambda + \mu \left(1 - \frac{f(x;L)}{f(x;H)}\right) \tag{3.4}$$

where $\lambda > 0$ and $\mu > 0$.[9]

(3.4) states that the optimal sharing rule is a positive affine function of the likelihood ratio $(f(x;H)/f(x;L))$ (Milgrom (1981)). We now need a:

**Definition 5 (Milgrom)** *A family of densities* $\{f(\cdot;a)\}$ *has the* monotone likelihood ratio property *if for every* $x > x' \in \Re$ *and* $a > a' \in \Re$,

$$\frac{f(x;a)}{f(x;a')} > \frac{f(x';a)}{f(x';a')}$$

*holds.*

Monotonicity of the optimal incentive scheme holds when the likelihood ratio is mono-

[8]The Lagrangian for this problem is

$$\begin{aligned}
\mathcal{L}(t(x);H) &= \int (x - t(x)) f(x;H)dx \\
&\quad -\lambda \left[\bar{u} - \int (u(t(x)) - c(H))\, f(x;H)dx\right] \\
&\quad -\mu \left[\int (u(t(x)) - c(L))\, f(x;L)dx - \int (u(t(x)) - c(H))\, f(x;H)dx\right].
\end{aligned}$$

Pointwise maximization yields the desired result.

[9]The individual rationality constraint must be binding ($\lambda > 0$). For suppose it were slack: then the transfer could be reduced so that utility decreases by a constant in each state of nature without altering the agent's action choice. The incentive compatibility constraint must be binding ($\mu > 0$), for suppose to the contrary that $\mu = 0$. Then from (3.4) risk sharing is first-best (the agent is paid a constant), but the resulting action will be $L$. Assuming the principal wants to implement $a = H$, it follows that $\mu > 0$. The usual assumption that $F(x;H)$ dominates $F(x;L)$ in the sense of first-order stochastic dominance ensures that the principal indeed prefers to implement $a = H$.

tonic, that is when the monotone likelihood ratio (MLR) condition is satisfied (Milgrom (1981)). Little, however, can be said about the shape of the optimal sharing rule. In particular, there are no natural restrictions which result in a linear sharing rule.

Letting $g(H)$ denote the prior probability that the agent chooses $a = H$ and letting $g(H; x)$ be the corresponding posterior probability given the outcome of the agent's action, the likelihood ratio may be rewritten as

$$\frac{f(x; H)}{f(x; L)} = \frac{[1 - g(H)]g(H; x)}{g(H)[1 - g(H; x)]}.$$

That is, the agent is rewarded for favorable news ($g(H; x) > g(H)$) and punished for unfavorable news ($g(H; x) < g(H)$). A corollary of this result is that the optimal contract should only be made conditional on "new" information, i.e. on outcomes that change the posterior; that is, information should enter the optimal contract only up to a sufficient statistic, a result due to Holmstrom (1979).

Returning to the more general continuous case, but still assuming that $A \subset \Re$ and $\pi \equiv x$, the solution to the program (3.1)–(3.3) is much harder to characterize, due to the incentive compatibility constraint (3.3). A simple solution may be obtained if the constraint (3.3) can be replaced by the first-order condition

$$\int u(t(x)) f_a(x; a) dx - c'(a) = 0, \tag{3.5}$$

where $f_a(\cdot; \cdot)$ denotes, of course, the derivative with respect to action. In this case, as Holmstrom (1979) has shown, the optimal sharing rule satisfies

$$\frac{v'(x - t(x))}{u'(t(x))} = \lambda + \mu \frac{f_a(x; a)}{f(x; a)}, \tag{3.6}$$

where $f_a(x; a)/f(x; a)$ is the local counterpart of the likelihood ratio. To see why, we have the following:

**Proposition 6 (Milgrom)** *A family of densities $\{f(\cdot; a)\}$ has the monotone likelihood ratio property if and only if for every $a$, $f_a(x; a)/f(x; a)$ is increasing in $x$.*

**Proof.** Note $\frac{f_a}{f} = \frac{\partial}{\partial a} \ln f$. Therefore, $\forall a', a''$,

$$f(x;a')/f(x;a'') = \exp\left(-\int_{a'}^{a''} (f_a(x;a)/f(x;a)da)\right).$$

The right hand side is increasing in $x$ if and only if the left hand side is increasing in $x$. ■

If the principal is risk neutral ($v''(\cdot) = 0$), a simple argument (due to Jewitt (1988)) establishes that $\mu > 0$. In the more general case, signing this Lagrange multiplier is much harder (cf. the discussion in Jewitt (1988, p. 1180n)).

**Lemma 7 (Jewitt)** *Given $v''(\cdot) = 0$, and placing the standard assumptions on the derivatives of $u(\cdot)$ and $c(\cdot)$, any $\mu$ satisfying (3.5) and (3.6) is positive.*

**Proof.** By some positive affine transformation, $v'(\cdot) = 1$. Substituting (3.6) into (3.5), we have

$$\int u(t(x))\left(\frac{1}{u'(t(x))} - \lambda\right) f(x;a)dx - \mu c'(a) = 0.$$

Integrating both sides of (3.6), and noting that the identity $\int f(x;a)dx \equiv 1$ implies

$$\frac{\partial}{\partial a}\int f(x;a)dx = 0,$$

or

$$\int f_a(x;a)dx = 0,$$

we obtain

$$\int \frac{1}{u'(t(x))} f(x;a)dx = \lambda.$$

The covariance of $u(t(x))$ and $\frac{1}{u'(t(x))}$ is thus equal to $\mu c'(a)$.[10] By assumption, $c'(\cdot) > 0$, and since $u$ and $\frac{1}{u'}$ are monotone in the same direction, $\mu \geq 0$. If $\mu = 0$, $t(x) = const.$, so that (3.5) fails. Therefore $\mu > 0$. ■

[10]Note that, for any random variable $X$, $cov(a(X), b(X)) = E\left[a(X)\left(b(X) - E(b(X))\right)\right] - E\left[E(a(X)(b(X) - E(b(X))\right]$. But of course $E\left[E(a(X)(b(X) - E(b(X))\right] = 0$.

However, replacing (3.3) by (3.5), which has come to be known as the "first-order approach," is not in general admissible. Rogerson (1985) presents conditions (specifically, MLR and the convexity of distribution function (CDF) condition) under which the first-order approach is valid. Jewitt (1988) proves sufficiency of the first-order approach under more palatable conditions.

Although Holmstrom's sufficient statistic result holds for the more general case, this is about all that can be said about the problem in general. Even such supposedly basic properties of the sharing rule as monotonicity can no longer be guaranteed.[11] Generally, the optimal contract can only be shown not to be declining everywhere and not everywhere to be increasing faster than the principal's payoff (Grossman and Hart (1983)).

Holmstrom's sufficiency results can be extended to a multi-agent setting (cf. Holmstrom (1982)).[12] In particular, Holmstrom shows that, if and only if there exists a sufficient statistic for output (or the observable outcome), each agent's contract should be based solely on that statistic. A corollary of this result is the importance of relative performance evaluation. When individual agents' outputs are not independent (for instance, the technology with which output is produced depends on a common, as well as an individual, uncertainty parameter) then agents' transfer payments should depend on individual, as well as group (or team), output. In this case, letting the payment schedule depend on peer performance allows extraction of information about the common uncertainty.

### 3.2.2 Intertemporal Incentives and Linear Incentive Contracts

The fact that even very simple economic settings result in highly nonlinear sharing rules is at odds with the casually empiricist observation that linear incentive contracts seem prevalent. Furthermore, from a modelling point of view, nonlinear sharing rules are difficult to work with. Holmstrom and Milgrom (1987) show that giving the agent an action space that is richer than the (essentially one-dimensional) choice of a function through the probability simplex over outcomes (as in the usual parameterized distribution function approach), may

[11] If the first-order approach is valid, MLRC is sufficient to guarantee monotonicity. If the first-order approach is not valid, either both MLRC and CDFC or a condition known as the "spanning condition" (Grossman and Hart (1983)) are needed to guarantee monotonicity.

[12] Note that in a multi-agent setting, moral hazard may arise even when agents are risk neutral. This is the case when individual agents' contributions to output (or the observable outcome) cannot be observed with certainty.

result in "linear" sharing rules. The multi-dimensionality they introduce is the choice of actions in an intertemporal setting: In each period, the agent (at a cost) chooses a probability distribution over outcomes. The agent's strategy is therefore the process $\{p_t\}$.[13]

The model is based on the parameterized distribution function approach. We begin here with a discrete outcome space, but the extension to continuous outcomes is (at least conceptually) straightforward. There are $T$ time periods. In period $t$, the agent chooses a probability distribution $p_t = (p_t^1, ..., p_t^N) \in P \subset \Delta^N$ over $N$ possible (publicly observable) outcomes $x_t^1, ..., x_t^N$, and corresponding profits for the principal, $\pi_t^1, ..., \pi_t^N$, at cost $c(p_t)$. Denote the outcome of the agent's choice of $p_t$ as $x_t$, and the profit to the principal as $\pi_t$. As before, and for expositional clarity only, we assume $\pi_t = x_t$, that is, the principal's payoff is defined over observable outcomes.

Importantly, the agent observes $X_{t-1} = (x_1, ..., x_{t-1})$ before choosing $p_t$ (she does not forget), so that her strategy is the process $\{p_t(X_{t-1})\}$. Crucially also, principal and agent have an exponential (constant absolute risk aversion) utility function, so that $u(m) = -\exp(-rm)$, and $v(m) = -\exp(-Rm)$, where $r$ and $R$ are the constants of absolute risk aversion for the agent and the principal, respectively.[14] Both principal and agent care only for the level of their final wealth after period $T$, which is $t(X_T) - \sum_{i=1}^T c(p_i)$ for the agent, and $\sum_{i=1}^T \pi_i - t(X_T)$ for the principal. The incentive scheme $t(X_T)$ is, of course, dependent on the entire history of the outcome process.

The separability bought by CARA utility (no wealth effects) and the knowledge of all previous outcomes at the time of making the next action choice, allow the modeler to view each period-$t$ decision problem as separate (and, up to differences in wealth, identical). In each period, the agent is therefore paid a wage to ensure participation, and a premium if the good outcome is observed. Overall, the incentive scheme rewards for the total number

[13] For expositional simplicity, we outline the discrete time version of the problem.

[14] Note that CARA implies that wealth effects can be factored out in the following way:

$$u(w + \bar{w}) = -\exp(-r(w + \bar{w})) = -(-\exp(-rw))(-\exp(-r\bar{w})) = -u(w)u(\bar{w}),$$

which implies, for instance, that if the individual rationality constraint holds for some outside option (certain equivalent), it also holds for any other outside option certain equivalent $\bar{w}$. That is, if $\int u(x)f(x)dx \geq u(0)$, then

$$\int u(x + \bar{w})f(x)dx = -u(\bar{w}) \int u(x)f(x)dx \geq -u(\bar{w})u(0) = u(\bar{w})$$

Similarly, it can be shown that wealth effects have no effect on incentive compatibility constraints.

of "successes"—a "linear" reward schedule.

To build intuition, assume initially that the technology that the agent operates is governed by a Bernoulli process (as in Holmstrom and Milgrom (1988)) and the agent controls the probability of success; that is, with probability $p_t$ the outcome in period $t$ is $x_t = 1$, and with probability $1 - p_t$, the period-$t$ outcome is $x_t = 0$. We will later generalize to a multinomial process. Assume also that there are only two periods, $T = 2$. Finally, let the principal be risk-neutral ($R = 0$). None of these assumptions are crucial in a general formulation of the model.

In period 1, the agent chooses $p_1$, and either of the two outcomes $\{0, 1\}$ realizes. In period 2, the agent chooses $p_{i2}$ (where $i = 0, 1$ indicates the period-1 outcome), and either of the two outcomes $\{0, 1\}$ realizes. The agent obtains reward $t_{11} \equiv t(\{1,1\})$ with probability $p_1 p_{12}$, reward $t_{10} \equiv t(\{1,0\})$ with probability $p_1(1 - p_{12})$, and so forth. Now write the principal's problem as:

$$\begin{aligned}\max_{p_1, p_{12}, p_{02}, t_{11}, t_{10}, t_{01}, t_{00}} & p_1 \left[p_{12}\left(2 - t_{11}\right) + \left(1 - p_{12}\right)\left(1 - t_{10}\right)\right] + \\ & +(1 - p_1)\left[p_{02}\left(1 - t_{01}\right) + \left(1 - p_{02}\right)\left(0 - t_{00}\right)\right]\end{aligned} \tag{3.7}$$

s.t.

$$\begin{aligned} & p_1 p_{12} u\left(t_{11} - c(p_1) - c(p_{12})\right) + p_1(1 - p_{12}) u\left(t_{10} - c(p_1) - c(p_{12})\right) + \\ & (1 - p_1) p_{02} u\left(t_{01} - c(p_1) - c(p_{02})\right) + (1 - p_1)(1 - p_{02}) u\left(t_{00} - c(p_1) - c(p_{02})\right) \\ \geq\ & \underline{u} = u(0) \end{aligned} \tag{3.8}$$

$$p_1 = \arg\max_{\tilde{p}_1} \tilde{p}_1 u\left(W_1(\cdot) - c(\tilde{p}_1)\right) + (1 - \tilde{p}_1) u\left(W_0(\cdot) - c(\tilde{p}_1)\right) \tag{3.9}$$

$$p_{i2} = \arg\max_{\tilde{p}_{i2}} \tilde{p}_{i2} u\left(t_{i1} - c(\tilde{p}_{i2})\right) + (1 - \tilde{p}_{i2}) u\left(t_{i0} - c(\tilde{p}_{i2})\right), \text{ for } i = 0, 1 \tag{3.10}$$

where $W_i(p_{i2}, t_{i0}, t_{i1})$ is defined as the solution to

$$u(W_i(\cdot)) = p_{i2} u\left(t_{i1} - c(p_{i2})\right) + (1 - p_{i2}) u\left(t_{10} - c(p_{i2})\right)$$

i.e. the certain equivalent of the period-2 lottery when action $p_{i2}$ is taken, given sharing rule $(t_{i0}, t_{i1})$.

By normalization of the agent's outside utility in terms of zero certain equivalent, (3.8) is the individual rationality constraint, (3.9) is the period-1 incentive compatibility constraint, and (3.10) are the period-2 individual rationality constraints, one for each period-1 outcome.

From (3.9) (the period-1 IC constraint), $p_1$ is determined by the difference $W_1(\cdot) - W_0(\cdot)$ (an implication of CARA). The principal therefore ensures first-period incentives through appropriate choice of this difference; the levels are pinned down by (3.8), which binds at the optimum. The important feature of CARA is therefore that the principal can treat the two constraints (3.8) and (3.9) separately.

Next, consider the following set of auxiliary (period-2) problems, supposing that $W_1(\cdot)$ and $W_0(\cdot)$ are exogenously chosen (at $w_1$ and $w_0$, respectively) to provide the right (period-1) incentives. The principal seeks to

$$\max p_{i2}\,(x_1 + 1 - t_{i1}) + (1 - p_{i2})\,(x_1 + 0 - t_{i0}) \tag{3.11}$$

s.t.

$$W_i(p_{i2}, t_{i0}, t_{i1}) = w_i \tag{3.12}$$

and (3.10).

The set of equations (3.12) are IR constraints, ensuring that the agent's expected utility in each period-2 problem is (at the optimum) just large enough to provide the desired (period-1) incentives, that is such that

$$u(W_i(p_{i2}, t_{i0}, t_{i1})) = p_{i2}u\,(t_{i1} - c(p_{i2})) + (1 - p_{i2})u\,(t_{i0} - c(p_{i2})) = u(w_i).$$

Just as in the period-1 problem, in the auxiliary problem, $p_{i2}$ is determined by the difference $t_{i1} - t_{i0}$ (through (3.10)), and $t_{i0}$ is chosen so as to ensure participation (3.12). The difference $t_{i1} - t_{i0}$, as well as $p_{i2}$, is therefore independent of $w_i$ (again an implication of CARA). An important implication of this independence is that the auxiliary problems for $i = 0$ and $i = 1$ are of the same form, so that in a solution to the principal's problem,

$t_{11} - t_{10} = t_{01} - t_{00}$. This, of course, implies that $p_{02} = p_{12}$.

We now need to ensure that $W_1(p_{12}, t_{10}, t_{11}) - W_0(p_{02}, t_{00}, t_{01}) = w_1 - w_0$, as required by (3.12). It is necessary (and sufficient) therefore to have $w_1 - w_0 = t_{10} - t_{00}$.[15] Since we know that $t_{11} - t_{10} = t_{01} - t_{00}$, this implies that $w_1 - w_0 = t_{11} - t_{01}$.

Considering now the period-1 problem, it is straightforward that since the period-1 problem and each period-2 problem are technologically identical (up to wealth differences) we must have $p_1 = p_{i2}$ and $w_1 - w_0 = t_{11} - t_{10} = t_{01} - t_{00}$.

Now define $\alpha$ as the reward for success in period 2 when the agent was unsuccessful in period 1: $\alpha = t_{01} - t_{00}$. But from $t_{11} - t_{10} = t_{01} - t_{00}$ we know that this is also the reward for success in period 2 when the agent was successful in period 1. Furthermore, since $t_{11} - t_{10} = t_{01} - t_{00} = w_1 - w_0 = t_{10} - t_{00} = t_{11} - t_{01}$, we also know that this has to be the reward for success in period 1 when the agent is either unsuccessful or successful in period 2. We therefore have

$$t_{01} = t_{10} = t_{00} + \alpha$$

and

$$t_{11} = t_{00} + 2\alpha.$$

In other words, the agent is rewarded purely on the number of successes, regardless of when they occur. In a $T$-period model, define $X$ as the number of successes, i.e. $X = \sum_{i=1}^{T} x_i$. The incentive scheme only needs to be based on that statistic; that is, it is of the form $t(X_T) = t(X) = \alpha X + \beta$.

In a more general discrete time model, the agent may be conceptualized as operating a multinomial process with $J$ possible different outcomes in each period. The model can

[15] Note that, since $p_{02} = p_{12}$, we can write $c(p_{02}) = c(p_{12}) = c$.

$$u(w_1 - w_0) = -u(w_1)u(-w_0) = -\frac{pu(t_{11} - c) + (1-p)u(t_{10} - c)}{pu(t_{01} - c) + (1-p)u(t_{00} - c)}.$$

CARA utility allows us to factor out $c$; noting further that $t_{11} - t_{10} = t_{01} - t_{00}$, we can expand the above expression by $\frac{u(t_{10})u(t_{01})}{u(t_{01})u(t_{10})}$ to obtain

$$u(w_1 - w_0) = u(t_{10} - t_{00}).$$

easily, and with only minor modifications, be extended to this case. Again, the optimal incentive scheme is of the form $t(X_T) = t(X(j)) = \sum_{j=1}^{J} \alpha_j X(j) + \beta$, where $X(j)$ is the number of times that outcome $j$ occurs.

A cleaner analysis obtains when the length of each period goes to zero. This is the model in Holmstrom and Milgrom (1987). The agent controls the constant drift rate of a (one-dimensional) Brownian motion process over the unit time interval, and the discrete-case Bernoulli assumption approximates to a normal distribution over the number of successes in the continuous case. Similarly, in the limit, the multinomial discrete case converges to a multi-dimensional Brownian motion process with the agent controlling the drift vector.

The continuous case approximation therefore motivates the following reduced form model: The agent, with utility $u(m) = -\exp(-rm)$, chooses action $a$, where output is related to action according to the technology $x = a + \theta$, with $\theta \sim N(0, \sigma^2)$, and contracts are restricted to the linear form $t(x) = \alpha x + \beta$. We use this linear model to build intuition for the common agency case. First, however, we study the linear model in a single-principal single-agent framework.

Note that the agent's objective may be written in terms of the agent's certain equivalent, so that she seeks to[16,17]

$$\max \alpha a + \beta - \frac{1}{2} r \alpha^2 \sigma^2 - c(a). \tag{3.13}$$

At any solution to the principal's program, therefore, the agent chooses her action such that

[16]The certainty equivalent ($CE$) for $\theta \sim N(\mu, \sigma^2)$ is calculated as follows:

$$\begin{aligned}
\exp(-rCE) &= \int \exp(-r(\alpha a + \alpha\theta + \beta)) \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(\theta-\mu)^2}{2\sigma^2}) d\theta \\
&= \exp(-r(\alpha a + \beta + \alpha\mu)) \int \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-r\alpha(\theta-\mu)) \exp(-\frac{(\theta-\mu)^2}{2\sigma^2}) d\theta \\
&= \exp(-r(\alpha a + \beta + \alpha\mu - \frac{1}{2} r\alpha^2\sigma^2)) \int \frac{1}{\sqrt{2\pi\sigma^2}} \exp(\frac{-\sigma^4 r^2 \alpha^2 - 2\sigma^2 r\alpha(\theta-\mu) - (\theta-\mu)^2}{2\sigma^2}) d\theta \\
&= \exp(-r(\alpha a + \beta + \alpha\mu - \frac{1}{2} r\alpha^2\sigma^2)) \int \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(\theta-\mu+\sigma^2 r\alpha)^2}{2\sigma^2}) d\theta
\end{aligned}$$

but the integral is just over a normal density with mean $\mu - \sigma^2 r\alpha$, so that we have

$$\exp(-rCE) = \exp(-r(\alpha a + \beta + \alpha\mu - \frac{1}{2} r\alpha^2\sigma^2)).$$

[17]This example follows Holmstrom and Milgrom (1988).

$\alpha = c'(a)$.

Similarly, the principal's objective may be written in terms of her certain equivalent (recall that the principal is assumed risk-neutral),

$$(1-\alpha)a - \beta. \tag{3.14}$$

Since we know that the outcome of contracting will be Pareto optimal (the principal maximizes utility subject to the agent's utility being maximized), the Pareto optimal allocation will maximize the sum of (3.13) and (3.14), that is

$$\max a - \frac{1}{2} r\alpha^2\sigma^2 - c(a) \tag{3.15}$$

s.t. $\alpha = c'(a)$.

To build intuition, let the agent have the following cost structure: $c(a) = \frac{1}{2}a^2$. In this case the solution to program (3.15) has $\alpha = \frac{1}{1+r\sigma^2}$.

The solution implies that, as our earlier discussion has suggested, the first-best solution (risk-neutral agent, $r = 0$, or no uncertainty, $\sigma^2 = 0$) has the agent own the technology ($\alpha = 1$). In second-best, the agent's share of profits is greater the lower her risk aversion, or the lower the risk in the technology the agent operates.

It is a useful exercise to consider the case in which the agent has two tasks. These results will provide the (second-best) benchmark case for our discussion of common agency, below.

In slight abuse of earlier notation, let $x_i$ denote the outcome in task $i$, and let $a_i$ denote the agent's action (or effort) in task $i$. Assume that the technology the agent controls is of the following simple additive and separable form: $x_i = a_i + \theta_i$, for $i = 1,2$, where the $\theta_i$ are drawn from a bivariate normal distribution. Given that the optimal linear scheme is $t(x_1, x_2) = \alpha_1 x_1 + \alpha_2 x_2 + \beta$, the agent seeks to maximize her certain equivalent,

$$\max CE(\alpha, a) \equiv \alpha_1 a_1 + \alpha_2 a_2 + \beta - \frac{1}{2} r \left(\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\sigma_{12}\right) - c(a), \tag{3.16}$$

where $\sigma_{12}$ denotes $cov(\theta_1, \theta_2)$. In any solution to the principal's problem, therefore, the agent will choose her action vector $a$ such that $\alpha_i = c_i(a) \equiv \frac{\partial}{\partial a_i}c(a)$, for $i = 1,2$. Again, the Pareto optimal solution maximizes the sum of the principal's and the agent's surplus, subject to $\alpha_i = c_i(a)$, $i = 1, 2$, so that the problem is to

$$\max CE(\alpha, a) + \sum(1 - \alpha_i)a_i, \tag{3.17}$$

s.t. $\alpha_i = c_i(a)$.

The solution to this program has

$$\frac{\partial CE}{\partial \alpha_i} - a_i + \sum(1 - \alpha_j)\frac{\partial a_j}{\partial \alpha_i} = 0, \text{ for } i = 1, 2$$

or written out

$$\sum_j (1 - \alpha_j)\frac{\partial a_j}{\partial \alpha_i} = r\left(\alpha_i \sigma_i^2 + \alpha_j \sigma_{12}\right), \text{ for } i = 1, 2 \tag{3.18}$$

where $\sigma_{12}$ again denotes $cov(\theta_1, \theta_2)$. The agent's first-order condition $\alpha_i = c_i(a_1(\alpha_1, \alpha_2), a_2(\alpha_1, \alpha_2))$ implicitly defines functions $a_i(\alpha_1, \alpha_2)$, which we can differentiate to obtain

$$\frac{\partial a_i}{\partial \alpha_i} = \frac{c_{jj}}{c_{11}c_{22} - (c_{12})^2}$$

$$\frac{\partial a_j}{\partial \alpha_i} = -\frac{c_{ij}}{c_{11}c_{22} - (c_{12})^2}.$$

Define $\Delta = c_{11}c_{22} - (c_{12})^2$, so that the first-order condition (3.18) may be rewritten

$$(1 - \alpha_i)c_{jj} - (1 - \alpha_j)c_{ij} = r\Delta\left(\alpha_i \sigma_i^2 + \alpha_j \sigma_{12}\right), \text{ for } i = 1, 2. \tag{3.19}$$

Below, we will use this solution to program (3.17) as the second-best benchmark for comparison with the non-cooperative provision of incentive schemes by several principals. The comparison is natural: (3.19) can be viewed as giving the optimal incentive scheme when two principals co-ordinate in their choice of transfer schedules.

Note also that if $c_{12} = 0$ and $\sigma_{12} = 0$ (activities are technologically and stochastically independent), $\alpha_1$ and $\alpha_2$ are independent of each other. Again, if the agent is either risk neutral or there is no uncertainty, the optimal incentive scheme has the agent owning the technology ($\alpha_i = 1$). If there is no uncertainty about one of the outcomes, in general the principal may still wish to provide incentives on that activity because it changes the agent's opportunity cost for choice of action in the other activity (so $\sigma_1 = 0$ need not imply $\alpha_1 = 1$, but will imply this if $\sigma_2 = 0$ also).

## 3.3 Common Agency under Moral Hazard

In this section, we study the available common agency models under conditions of moral hazard. To build intuition, we begin with the common agency model in Holmstrom and Milgrom (1988). The model uses linear incentive contracts, in the spirit of Holmstrom and Milgrom (1987), which simplifies much of the analysis. We then turn to the more general non-linear contracts case studied by Bernheim and Whinston (1986a).

### 3.3.1 The Case of Linear Incentive Contracts

This section analyzes common agency in the Holmstrom and Milgrom (1987) linear contracts framework. We continue with the same notation as above. However, our primary interest is now a description of the (Nash) equilibria achievable in the game amongst principals. Two principals non-cooperatively design incentive schemes for a common agent, who operates two technologies or "tasks," $i = 1, 2$. As is standard, the agent decides privately on the amount of effort she spends in each task. Each principal $i$ has an interest only in the agent's effort in task $i$.

Two situations are of interest, depending on what principals can observe. Holmstrom and Milgrom (1988) analyze the case of *disjoint observations* (each principal $i$ observes $z_i = x_i$), or *joint observations* (principal $i$ observes $z_i = (x_1, x_2)$).

#### Disjoint Observations

Under disjoint observations, principal 1 maximizes the joint surplus of principal 1 and the agent, that is she

$$\max S_1 \equiv a_1 + \alpha_2 a_2 - c(a) - \frac{1}{2} r \left(\alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2 + 2\alpha_1 \alpha_2 \sigma_{12}\right)$$

with the first-order condition

$$\frac{\partial a_1}{\partial \alpha_1} + \alpha_2 \frac{\partial a_2}{\partial \alpha_1} - c_1 \frac{\partial a_1}{\partial \alpha_1} - c_2 \frac{\partial a_2}{\partial \alpha_1} = r(\alpha_1 \sigma_1 + \alpha_2 \sigma_{12}).$$

A similar condition holds for the maximization of the joint surplus between principal 2 and the agent. Note that the agent's incentive compatibility constraint (first-order condition on action) is $c_i = \alpha_i$, so that the first-order condition for principal $i$'s problem becomes:

$$(1 - \alpha_i) \frac{\partial a_i}{\partial \alpha_i} = r\left(\alpha_i \sigma_i^2 + \alpha_j \sigma_{12}\right), \text{ for } i = 1, 2;\ j \neq i.$$

Using a Taylor approximation for the cost function, Holmstrom and Milgrom (1988) obtain the following approximation to the first-order condition:

$$(1 - \alpha_i) c_{jj} = r\Delta\left(\alpha_i \sigma_i^2 + \alpha_j \sigma_{12}\right), \text{ for } i = 1, 2;\ j \neq i$$

where $\Delta = c_{11}c_{22} - (c_{12})^2$. Compare this to the first-order condition in the second-best (co-operative) case:

$$(1 - \alpha_i) c_{jj} - (1 - \alpha_j) c_{ij} = r\Delta\left(\alpha_i \sigma_i^2 + \alpha_j \sigma_{12}\right), \text{ for } i = 1, 2;\ j \neq i$$

An obvious conclusion is that common agency does not introduce any additional distortion if the two activities are technologically independent, i.e. if $c_{12} = 0$. Intuitively, if one activity does not make the other more or less costly (at the margin), there is no room for contractual externalities between principals. Similarly, when $\alpha_i = 1$ for both activities (e.g. when the agent is risk-neutral or there is no uncertainty), there will be no harm from common agency.

Interestingly, consider the case where the two activities are stochastically independent ($\sigma_{12} = 0$). If the two activities are cost substitutes ($c_{12} > 0$), then incentive provision in common agency is greater than in second-best; similarly, if activities are complements ($c_{12} < 0$), then incentive provision in common agency is lower than in second-best. The intuition is that raising incentives in one activity leads the agent to work harder at the other activity if $c_{12} < 0$. This means that principals' incentive schemes may be less high-powered than in second-best.

### Joint Observations

Holmstrom and Milgrom (1988) next consider the case of joint observations, where each principal $i$ observes $z_i = (x_1, x_2)$. Each principal can now provide incentives conditional on performance in both tasks. Denote by $\alpha_{ij}$ the incentive provided by principal $i$ on $x_j$. The (total) incentive for the agent to perform task $j$ is therefore $\alpha_j = \alpha_{1j} + \alpha_{2j}$.

As before, we look for the Pareto-efficient incentive payments, that is we maximize the joint surplus of principals and agent. The joint surplus of the agent and principal 1 is

$$S_1 = (1 + \alpha_{21})a_1 + \alpha_{22}a_2 - c(a) - \frac{1}{2}r\left(\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\sigma_{12}\right),$$

and the joint surplus of the agent and principal 2 is

$$S_2 = \alpha_{11}a_1 + (1 + \alpha_{12})a_2 - c(a) - \frac{1}{2}r\left(\alpha_1^2\sigma_1^2 + \alpha_2^2\sigma_2^2 + 2\alpha_1\alpha_2\sigma_{12}\right).$$

Of course in equilibrium both principals will wish the same aggregate incentives, so that we must have $1 + \alpha_{21} = \alpha_{11}$ and $\alpha_{22} = 1 + \alpha_{12}$, so that (using $\alpha_j = \alpha_{1j} + \alpha_{2j}$) we have $\alpha_{ii} = \frac{1+\alpha_i}{2}$, and $\alpha_{ji} = \frac{\alpha_i - 1}{2}$. This implies that in equilibrium, each principal provides a high-powered incentive scheme on the performance in the task that enters that principal's payoff function, but each principals offers a negative incentive payment for the other task. In effect, principals use the agent to effect side payments to each other via the agent. Using these incentive shares in the surplus expressions, and maximizing, Holmstrom and Milgrom (1988) obtain the following characterization for equilibrium:

$$\sum_j (1 - \alpha_j)\frac{\partial a_j}{\partial \alpha_i} = 2r\left(\alpha_i\sigma_i^2 + \alpha_j\sigma_{12}\right), \text{ for } i = 1, 2$$

or again, using a Taylor approximation for cost,

$$(1 - \alpha_1)c_{22} - (1 - \alpha_2)c_{12} = 2r\Delta\left(\alpha_1\sigma_1^2 + \alpha_2\sigma_{12}\right)$$

$$-(1-\alpha_1)c_{12}+(1-\alpha_2)c_{11}=2r\Delta\left(\alpha_2\sigma_2^2+\alpha_1\sigma_{12}\right).$$

Again, comparing with the second best solution (3.19), aggregate shares are set as if each agent's constant of absolute risk aversion ($r$) were doubled. Naturally, this implies lower-powered incentives, and lower performance.

Holmstrom and Milgrom's results illustrate clearly the nature of the contractual externality in this common agency setting: two principals simultaneously provide incentives for an agent to perform two different tasks. When observations are disjoint, and the activities are technologically independent, non-cooperative contracting has no implication for output: there is no room for principals to impose externalities on each other. When tasks are related through the agent's cost function, we obtain an effect we also encounter in chapter 4: through their incentive-setting, principals impose externalities on each other. Suppose activities are substitutes: if principal 1 increases her incentive payment to the agent, the agent will work less hard at activity 2: a negative externality. This means that principal 2 will have to increase her incentive payment to the agent. The incentive scheme will be more high-powered than in (the co-operative benchmark case) second-best. When activities are complements, the opposite is true: since an increase in principal 1's incentive payment leads the agent to work harder at principal 2's activity, we have a positive externality, and lower than second-best incentive provision. With joint observations, there is a novel twist to this intuition: When principals can observe outcomes in both tasks, each principal can insure the agent against the risk imposed by the other principal in her incentive scheme. This leads to increased incentive shares paid by each principal on her activity. But since some of this is received by the other principal as a side payment (recall that with joint observations, $\alpha_{ii}>0$ and $\alpha_{ij}<0$), free riding results in an equilibrium level of incentive provision below the co-operative benchmark case.

A similar modeling exercise is conducted independently by Dixit (1996) and Dixit (1997). Dixit uses a multitask version of the linear contract model of Holmstrom and Milgrom (1987) to address the inefficiencies created in a common agency setting with multiple principals and joint observations. Except for the generalization to the case of $I$ principals, the model contains no new insights or indeed departures from the model in the unpublished paper by Holmstrom and Milgrom (1988). Unsurprisingly, Dixit finds that, if $I$ risk-neutral principals non-cooperatively provide incentives for a single common agent, the effect is the same as if the agent's constant of absolute risk aversion had been increased $I$-fold. This is just the $I$-principal generalization of the result Holmstrom and Milgrom (1988) obtained for the two-principal case, and which we have outlined above. Dixit rehearses the model to address the question of why incentives in government bureaucracies are typically weak; the obvious answer is that the non-cooperative behavior of several principals weakens the incentive scheme for the common agent.

After having built intuition in this simple linear contracting framework, we turn next to the more general model in Bernheim and Whinston (1986a).

### 3.3.2 The Case of Non-linear Incentive Contracts

The general theory of common agency under conditions of moral hazard is presented in Bernheim and Whinston (1986a). Several principals non-cooperatively offer a single agent incentive schemes under moral hazard. In their paper, common agency is intrinsic; i.e. the agent can participate in either all or none of the principals' incentive schemes.

The modelling approach is much like that in the single-principal single-agent case: an agent chooses unobservable actions, and is given incentive schemes (based on observable outcomes) by two principals. As is usual, for given incentive schemes the agent chooses the action that maximizes her utility (so the incentive schemes have to be incentive compatible), and she always has the option not to participate in any of the principals' mechanisms.[18] Now, however, the focus is not only on a description of the optimal incentive scheme and the implementable actions, but on a description of the equilibrium outcome in the incentive-setting game amongst principals. That is, each principal creates incentives for the agent, in competition with her rivals, but will only do so if she can guarantee herself an equilibrium payoff greater than that she could achieve by not interacting with the common agent.

[18]Note that the assumption of intrinsic common agency requires that the agent either partake in all or none of the principals' incentive schemes. Delegated common agency would allow the agent to accept some principals' incentive offers and not those of others, that is, it would introduce additional constraints on principals' actions.

## The Model

The model in Bernheim and Whinston (1986a) is a straightforward extension of the single-principal single-agent model, above.

Let there be $I$ risk-neutral principals $i$, who contract non-cooperatively with a single agent. The agent's action choice induces a distribution $p \equiv (p_1, ..., p_N)$ over the $N$-vector of possible outcomes for each principal $i$, $x^i \equiv (x^i_1, ..., x^i_N)$. Bernheim and Whinston model the problem using a discrete version of the general distribution function approach, so that the agent is modelled as choosing some probability distribution $p \in P \subset \Delta^N$ directly (where $\Delta^N$ is the $N$-dimensional unit simplex).

Each principal $i$ designs a transfer schedule (or sharing rule) $t^i \equiv (t^i_1, ..., t^i_N) \in \Re^N$, contingent only on observable (and verifiable) outcomes, but importantly each principal can observe (and condition her contract on) the entire probability distribution $p$, that is, the agent's contract actions in every principal's incentive scheme. Principal $i$'s expected utility given risk-neutrality is therefore $p \cdot (x^i - t^i)$, and her outside utility level is fixed at $\bar{x}^i$. The agent cares only about aggregate transfers $t \equiv \sum_{i=1}^{I} t^i$.

Denote the set of implementable distribution–transfer pairs $C \equiv \{(p, t) | p$ can be implemented by $t\}$. ($C$ could be defined by the conventional participation and incentive-compatibility constraints, or a more general constraint.) Principal $i$'s problem then is to

$$\max_{p, t^i} p \cdot (x^i - t^i) \tag{3.20}$$

s.t.

$$(p, t^i + \sum_{j=1, j \neq i}^{I} t^j_0) \in C, \tag{3.21}$$

where the subscript 0 denotes a principal's equilibrium choices.

In a non-cooperative equilibrium in this common agency, principals offer the agent optimal contracts given the contracting choices of the other principals, and each principal wishes to participate: each principal's expected utility has to exceed her reservation utility level, $\bar{x}^i$. We can therefore define non-cooperative equilibrium in this common agency game as the $I + 1$ tuple $(p_0, \{t^i_0\}_{i=1}^{I})$, where, for all $i = 1, ..., I$, the pairs $(p_0, t^i_0)$ are solutions to the maximization problem (3.20)–(3.21) and

$$p_0 \cdot (x^i - t_0^i) \geq \bar{x}^i \tag{3.22}$$

holds for all $i = 1, ..., I$.

Alternatively, by a change of variable, we can characterize an equilibrium $(p_0, \{t_0^i\}_{i=1}^I)$ as follows: for all $i = 1, ..., N$,

$$(p_0, t_0) \in \arg\max_{p,t} p \cdot (x^i + \sum_{j=1, j\neq i}^{I} t_0^j - t) \tag{3.23}$$

s.t.

$$(p, t) \in C \tag{3.24}$$

and such that

$$p_0 \cdot (x^i - t_0^i) \geq \bar{x}^i \tag{3.25}$$

holds. The interpretation of the equilibrium characterization (3.23)–(3.25) is that each principal offers her optimal contract by first undoing the other principals' incentive payments and then choosing an aggregate incentive scheme.

### An Alternative Equilibrium Characterization

Define $x \equiv \sum_{i=1}^I x^i$, and $\bar{x} \equiv \sum_{i=1}^I \bar{x}^i$. Bernheim and Whinston (1986a) show that, for some allocation of transfer payments $t_0^1, ..., t_0^I$ between principals, $(p_0, t_0)$ can arise in equilibrium if, and only if

$$(p_0, t_0) \in \arg\max_{p,t} p \cdot (x + (I-1)t_0 - It) \tag{3.26}$$

s.t.

$$(p, t) \in C \tag{3.27}$$

and such that

$$p_0 \cdot (x - t_0) \geq \bar{x} \tag{3.28}$$

holds. For a simple intuition, suppose $t_0^i = t_0^j$ for all $i, j = 1, ..., I$. Dividing the maximand in (3.26) by $I$, each principal seeks to maximize $\frac{x}{I} - \frac{t_0}{I} + t_0 - t$. That is, each principal gets an equal share of the net total monetary outcome $(\frac{x}{I} - \frac{t_0}{I})$, then undoes the other principals' incentive payments (by extracting from the agent $t_0$) and designs an aggregate incentive payment $t$. This intuition is evidently similar to the interpretation of the equilibrium characterization (3.23)–(3.25). The intuition can easily be generalized to any distribution of outcomes and incentive payments across principals. This equilibrium characterization later allows comparison with the cooperative solution.

### The Cooperative Benchmark Case

The cooperative outcome is of course isomorphic to the outcome in a single-principal single-agent problem where the principal chooses $(p_c, t_c) \in \arg\max_{p,t} p \cdot (x - t)$ s.t. $(p, t) \in C$.

### Properties of the Equilibrium

Two questions about non-cooperative equilibrium are immediate: First, what can be said about the properties of the aggregate equilibrium incentive scheme? From the equilibrium characterization (3.26)–(3.28) follows straightforwardly the following

**Proposition 8 (Bernheim and Whinston)** *If $(p_0, \{t_0^i\}_{i=1}^I)$ is an equilibrium, then the aggregate incentive scheme is the cost-minimizing one, i.e.* $t_0 \in \arg\min_t p_0 \cdot t$ *s.t.* $(p_0, t) \in C$.

**Proof. (sketch)** The interpretation of the equilibrium characterization (3.23)–(3.25) (and similarly, (3.26)–(3.28)) was that each principal may be viewed as undoing the other principals' contract offers and then offering an aggregate incentive scheme. A utility-maximizing principal will therefore, in equilibrium, choose the cost-minimizing aggregate incentive scheme. In this sense, the outcome of the common agency game is efficient. ■

Second, we are interested in the equilibrium agent action: How does the action that is implemented in a non-cooperative equilibrium compare to the cooperative benchmark?

Bernheim and Whinston (1986a) show that, in three special cases, the cooperative equilibrium can be achieved through non-cooperative contracting. Specifically, the cases Bernheim and Whinston (1986a) analyze are: (i) the cooperative incentive scheme $t_c$ is constant and the aggregate (non-cooperative) minimum cost incentive scheme that induces minimum effort from the agent, is constant; (ii) any agent action can be implemented at least cost by the same incentive scheme; (iii) under the standard principal-agent assumptions on $C$, the agent is risk-neutral. In any of these cases, the following holds:

**Proposition 9 (Bernheim and Whinston)** *In these special cases, the non-cooperative game will implement the cooperative (second best) action through the cost-minimizing (second best) incentive scheme* $(p_c, t_c)$.

The intuition for this result works along the following lines: consider the risk-neutrality case. If the agent is risk-neutral we know that the second-best incentive scheme is to sell the firm to the agent for a flat fee. In the common agency case, deviations from the cooperative benchmark are introduced through contract externalities that principals impose on each other. For instance, as we have seen in section 3.3.1 above, if one principal imposes a more high-powered incentive scheme, this will lead the agent to work harder in that principal's task; but if activities are related (say, through the agent's cost function), this has implications on the rival principal's desired incentive scheme. If the agent is risk-neutral, there must therefore exist a symmetric equilibrium, in which both principals use a constant incentive scheme. But by the previous proposition, we know that the aggregate incentive scheme has to be the cost-minimizing one.

This conclusion should not come as a surprise. In the linear contract case (section 3.3.1) the non-cooperative optimal incentive scheme was the cooperative incentive scheme for an agent twice as risk averse. Of course, if risk aversion is zero, the second-best incentive scheme coincides with the non-cooperative incentive scheme.

Note also that the conditions in proposition 9 are precisely the conditions under which the second-best and the first-best outcomes coincide.

In fact, there is more that can be said about the coincidence of non-cooperative and cooperative equilibrium actions and incentive schemes. The second-best optimal outcome can be achieved as an equilibrium outcome of the game amongst principals if, and only if, the conditions in proposition 9 hold. That is, when the first-best and second-best outcomes coincide, the non-cooperative outcome will also be second (and therefore, first) best. However, in all other cases, we have the usual inefficiency result: When first-best and second-best outcomes diverge, the non-cooperative equilibrium can never implement the second-best solution.

Although Bernheim and Whinston (1986a) do not prove existence of equilibria in the common agency game, they give sufficient conditions for existence. For instance, under the standard incentive-compatibility and participation constraint assumption on $C$, in the two-action case, with a strictly risk averse agent, an equilibrium will exist (but does not achieve the cooperative solution).

## 3.4 Applications

The following subsections discuss some straightforward applications of the general theory. Despite the frequency with which Bernheim and Whinston (1986a) are cited with reference to common agency type models, only few papers use the Bernheim and Whinston methodology of common agency under conditions of moral hazard. The few papers that are available are reviewed in what follows.

### 3.4.1 Common Sales Agency

Bernheim and Whinston (1998) study the use of common and exclusive sales agency contracts. One of the models developed in their paper is a straightforward application of the theory presented in Bernheim and Whinston (1986a).[19] A retailing agent chooses retail prices for the products of two independent manufacturers. Products are considered substitutes. The agent's price choice is unobservable by the principals, but linked to product sales through a stochastic technology. Manufacturers can base incentive schemes on observable sales, and Bernheim and Whinston assume that these incentive schemes are linear.

First, Bernheim and Whinston show that common or exclusive agency is chosen so as to maximize the total surplus available to manufacturers and agent. Further, as we have seen as

[19]Bernheim and Whinston (1998) also study models of exclusive dealing under symmetric information. Here, however, our focus is on common agency models under moral hazard.

a result of the general model above, when goods are substitutes, incentive provision will be inefficiently high: if manufacturer A reduces her incentive payment to the agent, the agent will reduce her selling effort for the good of manufacturer B also. In general, therefore, incentive provision will be inefficiently high, and the joint profits of manufacturers and agent will therefore be lower than in the second-best (cooperative) optimum. This creates an incentive for exclusive dealing: if the joint profits of manufacturers and retailer are higher under exclusive agency than under common agency, we would predict exclusive agency to arise. In fact, when products are perfect substitutes, all equilibria entail exclusive agency.

### 3.4.2 Competition for Regulatory Decision-Making

A further example of a common agency problem is that of interest groups competing for regulatory (pricing decision) favors.

Spiller (1990) models the situation where congress and an organized interest group ("industry") compete against each other through direct money payments to a price-regulating industry regulator. The regulator's privately observed action induces a probability distribution over prices; the distribution has support of size two, with the high price preferred by industry and the low price preferred by congress. Weighting the distribution towards the low price is costly for the regulator. The regulator is "bribed" in the following way: congress sets the regulatory budget contingent on the observed price; industry can effect direct money transfers to the regulator, conditional on observed price.

As we would expect, since regulator actions are technological substitutes (an increase in the action ("do nothing") favored by industry reduces the marginal cost of the action favored by congress ("do something")), the incentives given to the regulator are "too strong," that is, the monetary expenditure on regulation is greater than the minimum that is required to obtain the equilibrium regulatory outcome. Because of the negative externality each principal imposes on the other, each incentivizes "too much," relative to the co-operative outcome.

The empirical implication of Spiller's model is that since principals reward on success (in terms of their favored price outcome), we should expect to see regulators either remain within the public sector (and being promoted to "better" positions, or being given a larger budget), or (in the absence of easily effected monetary transfers from industry to regulators) leaving the public sector to work for the industry previously regulated by the regulator. On the whole (though not overwhelmingly), Spiller's data on the career paths of regulators confirms this prediction.

# Chapter 4

# Adverse Selection

This chapter considers adverse selection models of hidden information, that is, models in which the informational asymmetry is about a parameter in the agent's payoff (utility or cost) function. In the standard single-principal single-agent model, the principal designs a mechanism (or contract, or game) for the agent(s) to play. The mechanism design problem is then to set up the game so as to induce the agent to reveal (potentially through its actions, i.e. indirectly) its privately held information. For instance, in a simple non-linear pricing model where the agent knows privately a parameter of its utility function, the principal's problem is to design a mechanism (pricing schedule) so that each type of agent picks the price-quantity combination the principal has designed for that type. Generally this information-revelation mechanism comes at a cost to the principal: since the agent may gain by "lying", i.e. misrepresenting the true value of their privately known information, the mechanism needs to allocate some agent types informational rent in order to induce truth-telling (or truthful contract actions). The optimal mechanism balances the principal's gain from knowledge (i.e. being able to differentiate between the different agent "types") and the cost of allowing the agent to obtain informational rent. In the non-linear pricing example, the principal may want to sell more of a good to those agents who, at the margin, value the good more highly, but this needs to be balanced against the informational rent obtained by these agents, and the optimal contract may well (and generically, does) involve sub-optimal quantities (by comparison with the full-information case) being sold to agents with high (marginal) valuations for the good. The standard model therefore introduces the theme of an inefficiency: asymmetric information implies that rent be given to some agents; since rent is costly (and related to other contract variables, e.g. output in a non-linear pricing model), there is generally a distortion away from the full-information (first-best) optimum outcome.

In the common agency framework, a similar principle applies: each principal seeks to obtain information from agents in order to differentiate between them. The optimal contract again will involve some agents obtaining nonnegative rent. Now, however, the mechanism design problem not only is to balance the increased availability of information, and therefore increased benefit from differentiation between agents, against the amount of costly informational rent. In the common agency framework, principals are engaged in a mechanism design game amongst each other. Each principal designs its contract having to take into account that the agents' actions under its contract will in general depend on the other principals' contracts. If principals could find ways of costlessly coordinating actions, the second-best outcome from the single-principal single-agent model would naturally obtain. In many situations of economic significance, however, the mechanism design game between principals should be modelled as a non-cooperative game.[1] Since in general the agents' actions under each principal's mechanism depend on the incentives in the other principals' mechanisms (they may be substitutes or complements in the agents' payoff functions), each principal imposes a "contract externality" on other principals. It follows from a more general proposition about the sub-optimality of allocations when externalities are present, that the provision of incentives will either be greater or less than would be (second-best) optimal, depending on whether the principals' contracted variables are substitutes or complements in the agents' payoff functions. The common agency framework therefore introduces an additional, "contractual" inefficiency (i.e. additional to the simple rent-seeking inefficiency in the single principal model): since contract variables (e.g. outputs in a non-linear pricing model) may interact in the agents' payoff functions, the non-cooperative mechanism design leads to an "indirect" (with the agents' payoff functions as conduits) contractual inefficiency.

In terms of our (Coasean) organizing example, we might envision the question as this:

[1]Even if issues of commitment could be solved, there may exist institutional features which prevent coordination amongst principals. For instance, in a non-linear pricing example, antitrust law may prohibit principals from allowing the quantities sold to an agent to depend on the quantities sold by the other principals. Generically, non-cooperative modelling is called for when principals cannot contract on each other's contract variables (allocations to the agents).

the radio D.J. (the single agent) has private information about a market characteristic (for instance, about the requirements to market recordings successfully). The practice of record producers (the principals) to offer payola payments to the D.J. can, in light of our discussion, straightforwardly be interpreted as a mechanism to elicit the D.J.'s privately held information. But the problem quickly becomes more complex as we consider the nature of the D.J.'s private information. The market for records (as opposed to the market for airtime) is a market for differentiated goods. The D.J.'s knowledge of its audience may therefore be knowledge of a taste parameter that marks the anticipated popularity of any given recording. Importantly, though, this points to a deficiency in the theoretical common agency literature: The literature on common agency under conditions of adverse selection has, until very recently, assumed away the central issue of the problem in our motivating example. In the published common agency literature the only interaction between competing principals' contracts is through the indirect interaction of contract allocations in the agent's payoff function.[2] But this clearly is not the important focus for contractual externalities in the Coasean problem. There, the point of mechanism design for record producers is to obtain a larger share of airtime (if producer $A$'s record is played, there is less time for producer $B$'s record to be played) and, by implication, to sell records in a monopolistically competitive market with characteristics about which the producers are imperfectly informed. This, of course, is a serious drawback of the common agency literature, and may explain its comparatively low impact on applied work. We will return to this issue after an exposition of the common agency methodology as studied in the published literature.

## 4.1 Outline

To build intuition for the results under common agency, we begin by reviewing the single-principal single-agent version of the problem. First, we have to dispose of some preliminaries. In the simple single-principal single-agent context, a result is available that simplifies the principal's maximization problem in the adverse selection model considerably: the revelation principle, which we therefore review briefly in section 4.2. Furthermore, a central result in

[2] Very recently, the theoretical framework for studying questions of competition (through a common agent) between two producers of homogeneous goods in a duopolistic market has been studied in Martimort and Stole (1999a).

the analysis of adverse selection models with hidden information (the "constraint reduction theorem") requires an assumption often known as "Spence-Mirrlees single-crossing". Section 4.3 clarifies usage of single-crossing properties. In section 4.4 we analyze the single-principal single-agent adverse selection model. Finally, in section 4.5 we begin the analysis of the multi-principal single-agent version of the problem. Since the multi-agent context shifts the focus of analysis from straightforward maximization of the single principal's objective to the equilibrium of a game between mechanism designers, it turns out that straightforward applicability of the revelation principle to the multi-principal problem is not guaranteed. Before we consider the common agency version of the adverse selection model, we need to make this notion precise. This is the job of section 4.5.5. A closely related class of models studies the interaction of two principal-agent hierarchies, and we briefly review these models in section 4.6. Finally, section 4.7 considers applications of the general common agency model under adverse selection.

## 4.2 General Remarks I: The Revelation Principle

In its most general form, the revelation principle applies to very general mechanism design problems with a single principal and multiple agents (cf. Myerson (1982)). In such models, agents have private information both about their type and their actions. The single principal's (or, mechanism designer's) problem is the implementation of a mechanism that takes messages sent by the agents into (a probability distribution over) the set of messages from principal to agents (suggestions about agent actions) and the principal's decision (or, allocation). The principal chooses to implement the mechanism that gives her the highest level of expected utility.

The revelation principle (the term is due to Myerson (1981)) suggests that for any dominant strategy equilibrium[3] in this general mechanism, there exists a mechanism which is both direct and incentive compatible[4] that results in the same equilibrium outcome (Gibbard

[3] A revelation principle is also available for Nash implementation. We focus on dominant strategy implementation as the stronger equilibrium concept. Furthermore, in mechanism design problems with a single agent (which will be our focus), the two equilibrium conditions coincide.

[4] A mechanism is direct if, and only if, (i) each agents' message space coincides with her type space and (ii) the principal's message (or, strategy) space coincides with the set of possible agent decisions. A mechanism is incentive compatible (or, a revelation mechanism) if, and only if, in equilibrium, agents report their type truthfully and follow the principal's suggestion on their action. The following definitions make this precise.

(1973), Dasgupta, Hammond, and Maskin (1979), Myerson (1979), Myerson (1982)). In maximizing over mechanisms the principal can therefore restrict attention, without loss of generality, to direct revelation mechanisms. The revelation principle allows a simple description of the set of allocations that are achievable; and it simplifies maximization of the principal's objective.

To be specific, consider a less general situation in which agents possess private information only about their types, and focus attention on pure strategy implementation. This is the context in which we will use the revelation principle in this section. Let the set of agents be $\{j\}_{j=1,...,J}$. Each agent $j$ has preferences over a set of outcomes, $X$. Preferences are indexed by the agent's type $\theta_j \in \Theta_j$ and represented by the utility function $u_j(x, \theta_j)$. Let $\Theta \equiv \Theta_1 \times ... \times \Theta_J$. Let us define terms.[5]

**Definition 10** *A* (social) choice function *g is a function* $g : \Theta \rightarrow X$.

Let $s_j \in S_j$ be agent $j$'s strategy, define a strategy profile $s \equiv s_1 \times ... \times s_J$ and let $S \equiv S_1 \times ... \times S_J$, and define an outcome function $h : S \rightarrow X$.

**Definition 11** *A general* mechanism *with the exogenously determined strategy space $S$ is a "game form" (Gibbard (1973))* $\Gamma_S \equiv (S_1, ..., S_J, h)$.

We will often refer to mechanisms as "contracts." Note also that $\Gamma$ is a game form, not a game, since $\Gamma$ specifies outcomes, not payoffs. Payoffs depend, in addition, on the $\theta_j$.

**Definition 12** *The profile $s^*$ is a* dominant strategy equilibrium *of $\Gamma_S$ if, for all $j$, $\theta_j \in \Theta_j$, $s'_j \in S_j$ and $s_{-j} \in S_{-j}$:*

$$u_j(h(s_j^*(\theta_j), s_{-j}), \theta_j) \geq u_j(h(s'_j, s_{-j}), \theta_j).$$

**Definition 13** *The social choice function $g$ is* implementable in dominant strategies by $\Gamma_S$ *if there exists a dominant strategy equilibrium $s^*$ of $\Gamma_S$ such that $h(s^*(\theta)) = g(\theta)$ for all $\theta$.*

[5] In the entirety of this chapter we restrict attention to (economically natural) deterministic mechanisms (i.e. mechanisms that take messages into allocations, not into distributions over allocations). Furthermore, when we discuss common agency games among principals, we restrict attention to pure strategies: principals do not randomize over the contracts that they offer.

**Definition 14** *A* direct mechanism *is a mechanism* $\Gamma_\Theta \equiv (\Theta_1, ..., \Theta_J, \cdot)$, *i.e. a mechanism in which* $S_j = \Theta_j$ *for all* $j$.

**Definition 15** *The social choice function* $g$ *is* truthfully implementable in dominant strategies *if, for all* $j$, $s_j^*(\theta_j) = \theta_j$ *and* $s^*$ *is a dominant strategy equilibrium of the direct incentive-compatible (or, direct revelation) mechanism* $\Gamma_\Theta \equiv (\Theta_1, ..., \Theta_J, g)$.

The revelation principle then states that:[6]

**Proposition 16** *If there exists some mechanism* $\Gamma_S$ *that implements* $g$ *in dominant strategies, then* $g$ *is also truthfully implementable in dominant strategies.*

**Proof.** Suppose $\Gamma_S$ implements $g$ in dominant strategies using the strategy profile $s^*$. Then, by the definition of a dominant strategy equilibrium, for all $j$ and $s_j'$,

$$u_j(h(s_j^*(\theta_j), s_{-j}^*(\theta_{-j})), \theta_j) \geq u_j(h(s_j', s_{-j}^*(\theta_{-j})), \theta_j).$$

In particular, let $s_j' = s_j^*(\theta_j')$, so that

$$u_j(h(s_j^*(\theta_j), s_{-j}^*(\theta_{-j})), \theta_j) \geq u_j(h(s_j^*(\theta_j'), s_{-j}^*(\theta_{-j})), \theta_j).$$

By implementation, $g(\theta) = h(s^*(\theta))$, so we have

$$u_j(g(\theta_j, \theta_{-j}), \theta_j) \geq u_j(g(\theta_j', \theta_{-j}), \theta_j),$$

which proves the proposition. ■

The intuition for this result is simple. Imagine that each agent is asked to make a report about her type to an independent mediator that is committed to calculating outcomes according to the function $g \equiv h \circ s^*$. Since $s^*$ is the equilibrium report agents would choose under the outcome function $h$, there is no advantage to the agent in misrepresenting her type to the mediator.

[6]There exists a similar revelation principle for implementability in Bayesian Nash equilibrium. In the case where there is a single agent, the two equilibrium concepts coincide. Since the focus in what follows will be on single agent models, we present only the revelation principle for dominant strategy implementation.

Obviously, a decentralized version of the revelation principle also holds, in which the mechanism designer is restricted to the use of direct revelation mechanisms. By offering an appropriate direct mechanism (or, "contract") $\Gamma_\Theta$, the mechanism designer can achieve the same set of (dominant strategy) equilibria as that attainable under any general mechanism $\Gamma_S$. The revelation principle thus states that restriction to direct revelation mechanisms is without loss of generality: the set of equilibrium outcomes achievable under a general mechanism is no larger than that under direct revelation mechanisms.

Restricting attention to direct revelation mechanisms in the decentralized version of the revelation principle, however, is not without loss of generality in common agency games (cf. Martimort and Stole (1997), Martimort and Stole (1999b)). In particular, the allocations that are achievable by one principal will, in general, depend on the contracts offered by the other principals. Consequently, the focus is not (as in a single-principal single-agent problem) on straightforward optimization of a principal's objective, but on a description of the set of equilibria in the game between contract-setters. In this game, out-of-equilibrium messages may carry valuable information and may therefore sustain equilibria that cannot be achieved in a direct revelation game. This issue will be taken up in more detail in section 4.5.5.

## 4.3 General Remarks II: Single-Crossing Properties

Mechanism design relies on the assumption that agents can be "sorted" according to their hidden information parameter by offering them a schedule of options to choose from. This will only work if agents of a "higher" type (however defined) have higher marginal valuations (or higher marginal costs). The assumption is therefore sometimes referred to as a "sorting" condition, or, because of the authors who first made substantial use of the condition, the "Spence-Mirrlees single-crossing property." Mathematically, the condition is a special case of the more general supermodularity property of functions on a lattice, which has very natural interpretations in economics (cf. Athey, Milgrom, and Roberts (1996)). The following definitions and propositions connect several types of "sorting" conditions that are used in the literature. The purpose of this section is to clarify usage of single-crossing properties in economics, and the relations between several slightly different definitions that appear in the literature.

Let us first define terms.

**Definition 17** *Let $(X, \geq)$ be a lattice (i.e. the pair of set $X$ and the partial order $\geq$). Define $x \vee y \in X$ as the* join *of any two elements $x, y \in X$ if $x \vee y \equiv \inf\{z : z \geq x, z \geq y\}$. Let $x \wedge y$ be the* meet *of any two elements $x, y \in X$ if $x \wedge y \equiv \sup\{z : z \leq x, z \leq y\}$.*

**Example 18** *For $X = \Re^n$, and $\geq$ the componentwise order, $x \vee y = (\max(x_1, y_1), ..., \max(x_n, y_n))$ and $x \wedge y = (\min(x_1, y_1), ..., \min(x_n, y_n))$.*

**Definition 19** *The function $u : X \rightarrow \Re$ is* supermodular *if, for all $x, y \in X$,*

$$u(x \vee y) + u(x \wedge y) \geq u(x) + u(y).$$

The following theorem connects supermodularity and a notion of complementarity that we will use extensively in the following sections.

**Proposition 20 (Topkis)** *For $X = \Re^n \times \Re^m$, $\geq$ the componentwise order, and $u : X \rightarrow \Re$ twice continuously differentiable, $u$ is supermodular if and only if, for all $j = 1, ..., n$ and all $i = 1, ..., m$, $j \neq i$, and all $x$,*

$$\frac{\partial^2}{\partial x_j \partial x_i} u(x) \geq 0.$$

This proposition, proven in Topkis (1978) (but cf. also Milgrom and Shannon (1994)), thus allows supermodularity to be checked pairwise, rather than for all components of $x$ and $y$ at the same time.

The condition that the cross-partial of the agent's payoff function be nonnegative is often referred to as a "single-crossing property" in the mechanism design literature. The following definition and theorems explain why.

**Definition 21 (Milgrom and Shannon)** *The function $u : X \times \Theta \rightarrow \Re$ has the* single-crossing property in incremental returns *if for $x'' > x'$ and $\theta'' > \theta'$, $u(x'', \theta') > u(x', \theta')$ implies $u(x'', \theta'') > u(x', \theta'')$, and $u(x'', \theta') \geq u(x', \theta')$ implies $u(x'', \theta'') \geq u(x', \theta'')$. This property holds strictly if the second weak inequality holds strictly.*

Note that in the sections that follow, we will be concerned with objective functions of the form $u(x,\theta) + t(x)$. The following theorem relates single-crossing and supermodularity.

**Proposition 22** *The function $u(x,\theta)+t(x)$ has the single-crossing property in incremental returns for every $t$ if and only if $u$ is supermodular.*

Working with a more general function of the form $u(x, t(x), \theta)$, we have the following result:

**Proposition 23** *The function $u(x,t(x),\theta)$ has the single-crossing property in incremental returns for every $t$ if and only if, for $u_2 \neq 0$, $\frac{u_1}{|u_2|}$ is (weakly) increasing in $\theta$.*

These propositions are proven in Milgrom and Shannon (1994).

**Definition 24** *If $u$ satisfies the condition that $\frac{u_1}{|u_2|}$ be (weakly) increasing in $\theta$, $u$ is said to possess the* Spence-Mirrlees single-crossing property.

The reason this condition is referred to as a "single-crossing" property is that indifference curves of different types that satisfy this assumption only cross once: obviously, the marginal rate of substitution of good 1 for good 2 is $-\frac{u_2}{u_1}$. The single-crossing assumption then says that higher agent types have more steeply sloped indifference curves at every point. Naturally, this implies that these indifference curves may only cross once. Throughout, we will assume strict single-crossing properties.

We will also throughout make the (economically related) assumption that $\frac{\partial u(x,\theta)}{\partial \theta} > 0$. This is, of course, similar to the "sorting" condition: higher agent types have higher absolute (not just higher marginal) valuations.

These supermodularity conditions will play a central part in the proof of a characterization theorem (the "constraint reduction theorem") that allows us to simplify drastically the informational constraints on the principal's maximization problem. We now turn to a discussion of this.

## 4.4 The Single-Principal Single-Agent Adverse Selection Model

In a very general formulation of the adverse selection model with hidden information, the principal's objective is to design a mechanism that allows agents to send messages (drawn from an exogenously defined message space) to the agent, and which implements the principal's preferred allocation (her "(social) choice function" in the language of the preceding definitions).[7] In general, the optimal allocation will depend on the agent's message, which in turn depends on the agent's private information. By the revelation principle we can, without loss of generality, restrict attention to direct revelation mechanisms, i.e. mechanisms which allow only messages on the agent's private information space and which, in equilibrium, induce truthful revelation of the agent's information. Here, we consider mechanisms designed for a single agent. Multi-agent models introduce issues of coalition-formation which are largely orthogonal to our concerns.

In much of what follows it will be useful to keep a "non-linear pricing" model in mind. In a general (indirect mechanism) non-linear pricing model, the agent would send messages to the principal, who then determines an allocation $(x, t)$ (a pair of quantity and price) for the agent. The agent values higher quantities of $x$ more highly, and pays the principal a price $t$ for the quantity she receives. The revelation principle allows the principal to restrict consideration to direct revelation mechanisms, i.e. mechanisms in which the agent announces her type truthfully and the principal determines quantity-price pairs as functions of the agent's type.

In such a model, the revelation principle allows the principal to design a contract (or allocation) $\{(x(\hat{\theta}), t(\hat{\theta}))\}_{\theta\in\Theta}$ that induces the decision $x(\cdot)$ and determines a monetary transfer $t(\cdot)$ as functions of the agent's announcement $\hat{\theta}$ on her type. Here we assume that positive $t$ are payments from the principal to the agent, and negative $t$ are payments from the agent to the principal. We also assume throughout that $x(\cdot)$ is $C^1$, i.e. has a continuous first derivative. As before, the allocation has to be implementable (or "incentive compatible") and it has to satisfy the agent's participation constraint. Below, we specialize the general

[7] The classic references are: Mirrlees (1971), Mussa and Rosen (1978), Guesnerie and Laffont (1984). For textbook treatments, cf. Salanié (1997), Stole (1997).

definition of implementability made in section 4.2 to the present context. First, we need to introduce a little more notation.

Let there be an agent of type $\theta \in \Theta \subset \Re$ (for simplicity, let $\Theta = [\underline{\theta}, \overline{\theta}]$), where $\theta$ is distributed according to the distribution function $F(\theta)$ with strictly positive density $f(\theta)$. Restricting attention to direct mechanisms in which the agent makes an announcement $\hat{\theta}$ on her type, the principal's objective is to maximize $v(x(\hat{\theta})) - t(\hat{\theta})$. Let a type $\theta$ agent have the following (additively separable) utility function: $u(x, \theta) + t$; her indirect utility from announcing her type as $\hat{\theta}$ is therefore $U(\hat{\theta}, \theta) \equiv u(x(\hat{\theta}), \theta) + t(\hat{\theta})$. The standard approach to the problem also assumes suitable differentiability of $u$ and $t$.

Recalling the interpretation of the problem as a non-linear pricing game, we may interpret $u(x, \theta)$ as a type $\theta$ agent's utility from consuming quantity $x$, and $t$ is the price of consumption. Note that on this interpretation, $t < 0$, i.e. the transfer flows from agent to principal. (The same notation covers, for instance, a problem in which an agent produces an input into the principal's production, so that $u(x, \theta) < 0$ can be interpreted as the agent's cost of producing $x$, with cost parameter $\theta$, and with $t > 0$ the payment from principal to agent.)

We are now in a position to define the notions of implementability and feasibility for this game.[8]

**Definition 25** *An allocation* $(x(\cdot), t(\cdot))$ *is* implementable *(or $x$ is implementable by $t$) if there exists a transfer schedule $t(\cdot)$ such that the agent's incentive compatibility constraint is satisfied, that is, if for all $(\theta, \hat{\theta}) \in \Theta^2$*

$$u(x(\theta), \theta) + t(\theta) \geq u(x(\hat{\theta}), \theta) + t(\hat{\theta}),$$

*or, for all $\theta \in \Theta$,*

$$U(\theta, \theta) = \max_{\hat{\theta} \in \Theta} U(\hat{\theta}, \theta).$$

*(Making use, of course, of the revelation principle.)*

[8] We standardize the level of utility the agent could achieve by not participating in the principal's mechanism to zero.

**Definition 26** *In addition, an allocation* $(x(\cdot), t(\cdot))$ *is* feasible *if* $U(\theta, \theta) \geq 0 \; \forall \theta \in \Theta$ *(that is, if individual rationality or the "participation constraint" is satisfied).*

The solution follows a two-step procedure. First, the set of implementable allocations is characterized. Then, the principal chooses the preferred allocation from the set of feasible allocations.

### 4.4.1 Intuition: A Binary Model

To build intuition, we first consider a very simple model in which the principal designs a mechanism for an agent of type $\theta \in \{\underline{\theta}, \overline{\theta}\}$, $\underline{\theta} < \overline{\theta}$, with the distribution $\{(\underline{\theta}, p), (\overline{\theta}, (1-p))\}$. We modify the above assumptions suitably to this discrete case: The principal determines an allocation (which, by the revelation principle, depends only on the agent's type) $(\underline{x}, \underline{t})$ for a type $\underline{\theta}$ agent and $(\overline{x}, \overline{t})$ for a type $\overline{\theta}$ agent. The agent's utility function is quasilinear of the form $u(x, \theta) + t$, and the principal's utility (similarly quasilinear) is $v(x) - t$. The problem can be viewed as a non-linear pricing problem, in which an agent of type $\theta$ has higher marginal valuations of a good $x$. Note also that this interpretation requires $t < 0$, so that the money payments flow from agent to principal. In this model, the principal seeks to maximize her expected utility, that is,

$$\max_{\underline{x}, \overline{x}, \underline{t}, \overline{t}} p \left[v(\underline{x}) - \underline{t}\right] + (1-p) \left[v(\overline{x}) - \overline{t}\right],$$

subject to the allocations $\{(\underline{x}, \underline{t}), (\overline{x}, \overline{t})\}$ being implementable and feasible, according to the definitions above. In this context, implementability requires that the following set of two incentive compatibility constraints holds:

$$u(\underline{x}, \underline{\theta}) + \underline{t} \geq u(\overline{x}, \underline{\theta}) + \overline{t} \tag{4.1}$$

$$u(\overline{x}, \overline{\theta}) + \overline{t} \geq u(\underline{x}, \overline{\theta}) + \underline{t}. \tag{4.2}$$

Feasibility requires that the following set of individual rationality constraints holds:

$$u(\underline{x}, \underline{\theta}) + \underline{t} \geq 0 \tag{4.3}$$

$$u(\overline{x},\overline{\theta}) + \overline{t} \geq 0. \tag{4.4}$$

We first determine a characterization of implementable and feasible allocations.

Obviously constraint (4.4) is slack. The reasoning is that (4.2) implies that $u(\overline{x},\overline{\theta}) + \overline{t} \geq u(\underline{x},\overline{\theta}) + \underline{t} > u(\underline{x},\underline{\theta}) + \underline{t} \geq 0$. The second inequality follows from the assumption $u_\theta > 0$, and the final inequality is constraint (4.3).

Next, it is easy to prove that constraint (4.2) must bind. For if not, $\overline{t}$ could be lowered by $\varepsilon$. This would make (4.1) easier to be satisfied, and since (4.4) is slack, it would have no implication on feasibility. But since lowering $\overline{t}$ increases the principal's objective, it must be worthwhile. Therefore (4.2) must bind.

A similar argument establishes that (4.3) binds. For if not, $\overline{t}$ and $\underline{t}$ could both be reduced by $\varepsilon$. This would leave (4.1) and (4.2) unchanged and since (4.4) is slack, feasibility is not compromised. But since reducing $t$ increases the principal's objective, it would be worthwhile. Therefore (4.3) binds.

Finally, (4.1) is slack. Since (4.2) binds, we have $\overline{t} - \underline{t} = u(\underline{x},\overline{\theta}) - u(\overline{x},\overline{\theta})$, and from (4.1) we have $u(\underline{x},\underline{\theta}) - u(\overline{x},\underline{\theta}) \geq \overline{t} - \underline{t}$. Combining these two we obtain $u(\overline{x},\underline{\theta}) - u(\underline{x},\underline{\theta}) \leq u(\overline{x},\overline{\theta}) - u(\underline{x},\overline{\theta})$. We assume that $\overline{x} > \underline{x}$, and verify this assumption *ex post*. Single-crossing then implies that the inequality holds strictly.

The set of constraints (4.1) to (4.4) can therefore be reduced to the two constraints

$$\underline{t} = -u(\underline{x},\underline{\theta})$$

and

$$\overline{t} = -u(\overline{x},\overline{\theta}) + u(\underline{x},\overline{\theta}) - u(\underline{x},\underline{\theta}).$$

This implies that the $\underline{\theta}$ type agent's rent (the agent's equilibrium payoff when she tells the truth about her type) is $U(\underline{\theta},\underline{\theta}) = 0$, and the $\overline{\theta}$ type agent's rent is $U(\overline{\theta},\overline{\theta}) = u(\underline{x},\overline{\theta}) - u(\underline{x},\underline{\theta}) > 0$ (by the single-crossing assumption). Further, we can simplify the principal's objective to

$$\max_{\underline{x},\overline{x},\underline{t},\overline{t}} p\left[v(\underline{x}) + u(\underline{x},\underline{\theta})\right] + (1-p)\left[v(\overline{x}) + u(\overline{x},\overline{\theta}) - \left(u(\underline{x},\overline{\theta}) - u(\underline{x},\underline{\theta})\right)\right].$$

The first-order conditions are:[9]

$$v'(\underline{x}) + u_x(\underline{x}, \underline{\theta}) = \frac{(1-p)}{p}\left[u_x(\underline{x}, \overline{\theta}) - u_x(\underline{x}, \underline{\theta})\right]$$

and

$$v'(\overline{x}) + u_x(\overline{x}, \overline{\theta}) = 0.$$

Since $v'' < 0$, and by single-crossing, in relation to first-best, the optimal contract implies an extra distortion downward in the allocation of the $\underline{\theta}$ type agent. This, of course, also confirms our earlier assumption that $\overline{x} > \underline{x}$.

### 4.4.2 A Continuum of Types

We now study the problem with a continuum of types. We return to our earlier continuous-case assumptions on type distribution and preferences.

Consider first implementability (incentive compatibility). Since we can restrict attention to direct mechanisms, the agent chooses a report to $\max_{\hat{\theta}\in\Theta} U(\hat{\theta}, \theta)$. This defines her optimal report correspondence $\hat{\theta}(\theta)$. Define the agent's rent, as before, as the reduced-form maximum value function $U(\theta) \equiv U(\hat{\theta}(\theta), \theta)$. By the envelope theorem, we have

$$U'(\theta) = u_\theta(x(\hat{\theta}(\theta)), \theta). \tag{4.5}$$

Where $U'(\theta)$ denotes the total derivative of $U(\cdot)$ (i.e. $\frac{d}{d\theta}U(\theta)$), evaluated at $\theta$. Since by assumption $u_\theta > 0$, we find that the agent's rent is strictly increasing in type. This is a generalization of the result we have obtained in the discrete (two-type) model, above. From the revelation principle we know that, in equilibrium, $\hat{\theta}(\theta) = \theta$. Equation (4.5) is

[9] Note that the first-order conditions in the full information first-best case (where $\underline{t} = -u(\underline{x}, \underline{\theta})$ and $\overline{t} = -u(\overline{x}, \overline{\theta})$) are:

$$v'(\underline{x}_{FB}) + u_x(\underline{x}_{FB}, \underline{\theta}) = 0$$

and

$$v'(\overline{x}_{FB}) + u_x(\overline{x}_{FB}, \overline{\theta}) = 0.$$

By $v'' < 0$ and single-crossing, it follows that $\underline{x}_{FB} < \overline{x}_{FB}$.

therefore the necessary first-order condition for the agent's maximization problem, and can be expressed equivalently as:

$$u_x(x(\theta),\theta)x'(\theta) + t'(\theta) = 0 \tag{4.6}$$

with the second-order condition

$$u_{xx}(x(\theta),\theta)\left(x'(\theta)\right)^2 + u_x(x(\theta),\theta)x''(\theta) + t''(\theta) \leq 0 \tag{4.7}$$

Equation (4.6) is a constant function in $\theta$, so we can differentiate and substitute into (4.7) to obtain

$$u_{x\theta}(x(\theta),\theta)x'(\theta) \geq 0. \tag{4.8}$$

We have the following simple characterization theorem (Mirrlees (1971)):

**Proposition 27 ("Constraint Reduction Theorem")** ***Necessity:*** *A contract is implementable only if equations (4.6) and (4.8) are satisfied.* ***Sufficiency:*** *Assume that the equilibrium decision function is increasing (i.e. $x'(\cdot) > 0$), and assume further that $u(\cdot,\cdot)$ has the single-crossing property in incremental returns; i.e. when $u$ is suitably differentiable, $u_{x\theta} > 0$. Then, the allocation is implementable if the first-order condition (4.6) holds.*

**Proof. Necessity:** First, we must show that $t'(\cdot)$ exists. An argument from revealed preference establishes that

$$U(\theta+\Delta\theta,\theta+\Delta\theta) - U(\theta+\Delta\theta,\theta) \geq U(\theta+\Delta\theta,\theta+\Delta\theta) - U(\theta,\theta) \geq U(\theta,\theta+\Delta\theta) - U(\theta,\theta).$$

Dividing by $\Delta\theta$ and taking limits as $\Delta\theta \to 0$, we obtain

$$\frac{d}{d\theta}U(\theta,\theta) = U_\theta(\theta,\theta).$$

So we know that the total derivative $\frac{d}{d\theta}U(\theta,\theta)$ exists. Taking a Taylor expansion, we have

$$U(\theta+\Delta\theta,\theta+\Delta\theta) - U(\theta,\theta) = u_x(x(\theta),\theta)x'(\theta)\Delta\theta + \frac{t(\theta+\Delta\theta)-t(\theta)}{\Delta\theta}\Delta\theta + ...$$

Again, dividing by $\Delta\theta$, and taking limits as $\Delta\theta \to 0$, we have:

$$\frac{d}{d\theta}U(\theta,\theta) = u_x(x(\theta),\theta)x'(\theta) + t'(\theta).$$

Since $\frac{d}{d\theta}U(\cdot,\cdot)$, $u_x(\cdot,\cdot)$, and $x'(\cdot)$ exist, $t'(\cdot)$ exists, so that $t(\cdot)$ is $C^1$. Equations (4.6) and (4.8) are the standard first and second order conditions.

**Sufficiency:** Suppose, to the contrary, that there exists a $\hat{\theta} \neq \theta$, such that $U(\hat{\theta},\theta) > U(\theta,\theta)$. Since $U(\hat{\theta},\theta) = U(\hat{\theta},\hat{\theta}) + u(x(\hat{\theta}),\theta) - u(x(\hat{\theta}),\hat{\theta})$, we have

$$u(x(\hat{\theta}),\theta) - u(x(\hat{\theta}),\hat{\theta}) > U(\theta,\theta) - U(\hat{\theta},\hat{\theta})$$

or, taking integrals,

$$\int_{\hat{\theta}}^{\theta} u_\theta(x(\hat{\theta}),s)ds > \int_{\hat{\theta}}^{\theta} \frac{d}{d\theta}U(s,s)ds$$

and using equation (4.5),

$$\int_{\hat{\theta}}^{\theta} u_\theta(x(\hat{\theta}),s)ds > \int_{\hat{\theta}}^{\theta} u_\theta(x(s),s)ds.$$

Rewriting,

$$\int_{\hat{\theta}}^{\theta} [u_\theta(x(\hat{\theta}),s) - u_\theta(x(s),s)]ds > 0$$

we obtain a contradiction, because, by assumption, $u_{x\theta} > 0$. ■

Again, we normalize the agent's outside utility to zero, so that her participation constraint is $U(\theta) \equiv u(x(\theta),\theta) + t(\theta) \geq 0$.

The principal's problem is then to

$$\max_{x(\cdot),U(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} [v(x(\theta)) + u(x(\theta),\theta) - U(\theta)]\, f(\theta)d\theta$$

s.t. $U'(\theta) = u_\theta(x(\theta),\theta)$ and $U(\underline{\theta}) = 0$.[10]

[10] Since $U(\theta)$ is increasing, the participation constraint $U(\theta) \geq 0$ reduces to $U(\underline{\theta}) = 0$ (the principal minimizes costly rent).

We use linear programming techniques to find the solution to this maximization program. The Hamiltonian for this problem is[11]

$$H(x, U, \theta, \lambda) \equiv (v(x(\theta)) + u(x(\theta), \theta) - U(\theta))\, f(\theta) + \lambda(\theta) u_\theta(x(\theta), \theta).$$

By Pontryagin's principle, we have $\lambda'(\theta) = -\frac{\partial H}{\partial U} = f(\theta)$. Integrating from $\theta$ to $\overline{\theta}$ and respecting the transversality condition $\lambda(\overline{\theta}) = 0$ yields $\lambda(\theta) = -(1 - F(\theta))$. $\frac{\partial H}{\partial x} = 0$ then implies that the optimal allocation solves the familiar (e.g. Salanié (1997)) condition:

$$v_x(x(\theta)) + u_x(x(\theta), \theta) = \frac{1 - F(\theta)}{f(\theta)} u_{x\theta}(x(\theta), \theta), \tag{4.9}$$

where $\frac{1-F(\theta)}{f(\theta)}$, of course, is the inverse hazard rate. Again, the optimal contract distorts $x$ downward for all but the highest type (for $\theta = \overline{\theta}$, the right-hand side of (4.9) is zero, which is just the first-best full information outcome). This equilibrium can easily be interpreted as the familiar (from price theory) equality between marginal benefit and marginal cost: the marginal benefit of increasing $x$ is $v_x(x(\theta)) + u_x(x(\theta), \theta)$ and this occurs with probability $f(\theta)$; the marginal cost of increasing $x$ is that rent has to be increased (by $u_{x\theta}(x(\theta), \theta)$) for all types higher than $\theta$, and $1 - F(\theta)$ measures the proportion of types higher than $\theta$.

We still need to verify monotonicity of $x(\cdot)$, which we have previously assumed. This is now no longer automatic, but can be ensured by placing regularity conditions on the hazard rate and the utility functions above. If the hazard rate is monotone (increasing), $v_{x\theta}(\cdot) \geq 0$, and $u_{x\theta\theta}(\cdot) \leq 0$, the resulting equilibrium decision schedule $x(\cdot)$ is of course also monotone. This monotone hazard rate condition (MHRC) is fulfilled, for instance, by the uniform, normal, logistic and exponential distributions.

The optimal transfer schedule $t(\theta) = \int_{\underline{\theta}}^{\theta} u_\theta(x(s), s) ds - u(x(\theta), \theta)$ is then obtained from (4.6) by simple integration.

[11] Always assuming that $x'(\cdot) > 0$. If not, "bunching" (areas of pooling of types) may occur, which we conveniently ignore here. If $x(\cdot)$ is not increasing throughout, the (somewhat cumbersome) technique is to optimally "smooth out" the $x(\cdot)$ function to a nondecreasing function. Along the increasing portions of that smoothed function the previous analysis applies; on the constant portion of the smoothed function, bunching (i.e. pooling) of types occurs. For an algorithm for optimal smoothing of the decision function cf. Guesnerie and Laffont (1984).

### 4.4.3 The Taxation Principle

Let us come back briefly to the non-linear pricing example. It is easy to recast the allocation $\{(x(\theta), t(\theta)\}_{\theta\in\Theta}$ in terms of a nonlinear tariff. Note that the "decision function" $x(\theta)$ assigns a quantity to every agent type. Since it is strictly monotonic (by placing sufficient regularity conditions on utility functions and distribution), it can be inverted. The inverse function $x^{-1}(x)$ tells us, for any quantity, what type should optimally obtain that quantity. By substitution into the transfer function $t(\theta)$ we obtain $T(x) \equiv -t(x^{-1}(x))$. A result known as the "taxation principle" states that any such conversion can always be carried out. That is, any direct mechanism can always be converted into an (indirect) nonlinear pricing mechanism in this way (cf. Guesnerie (1995)). Note that we have defined the tariff schedule $T(x)$ so that it is positive, as $t(\theta)$ is negative in the setting of this problem.

From (4.6) we know that $u_x(x(\theta), \theta)x'(\theta) + t'(\theta) = 0$, so that $T'(x) = -\frac{t'(\theta)}{x'(\theta)} = u_x(x, x^{-1}(x))$. We are interested in the sign of $T''(x)$. Differentiating, we note that $T''(x) = u_{xx}(x, \theta) + u_{x\theta}(x, \theta)\frac{1}{x'(\theta)}$, so that concavity of the tariff ($T''(x) < 0$) reduces to the condition that $x'(\theta) < -\frac{u_{x\theta}(x,\theta)}{u_{xx}(x,\theta)}$. Differentiating (4.9) with respect to $\theta$, we obtain (the dependence of $x$ on $\theta$ is suppressed in notation):

$$x'(\theta) = \frac{u_{x\theta}(x, \theta) - \frac{d}{d\theta}\left(\frac{1-F(\theta)}{f(\theta)}\right) u_{x\theta}(x, \theta) - \frac{1-F(\theta)}{f(\theta)} u_{x\theta\theta}(x, \theta)}{-u_{xx}(x, \theta) + \frac{1-F(\theta)}{f(\theta)} u_{xx\theta}(x, \theta) - v_{xx}(x)}.$$

Since, by assumption, we have $u_{x\theta} > 0$, $\frac{d}{d\theta}\left(\frac{1-F}{f}\right) < 0$, $u_{xx\theta} > 0$, and $v_{xx} < 0$, and if we are willing to assume that $u_{x\theta\theta} < 0$, we obtain $x'(\theta) < -\frac{u_{x\theta}(x,\theta)}{u_{xx}(x,\theta)}$, which implies strict concavity of the non-linear pricing schedule, just as anticipated.

## 4.5 Common Agency under Adverse Selection

We now set up the non-cooperative mechanism design game amongst several principals, who contract with a single agent. We restrict attention to models with one agent and two principals. Increasing the number of principals does not materially alter the approach we take. The two-principal model allows us to study the additional externality that arises through non-cooperative contracting of the two principals, while keeping the analysis as uncluttered as possible.

The general theory is presented in Martimort (1992) and Stole (1992). Two principals contract with a common agent under adverse selection. The setup is a slight modification of the standard model outlined above. There are two important assumptions: (a) the agent announces her type separately to the principals; and (b) neither principal may condition her contract on the agent's message to the other principal. Part (a) is, as far as we can see, relatively uncontroversial. It implies that (although this will not occur in equilibrium) an agent could lie to one principal and tell the truth (or possibly lie differently) to the other principal. The assumption therefore states that announcements of type to one principal are not public, or if they are, they are made in a manner that is not verifiable (and so cannot be part of the other principal's contract). This also highlights the more critical importance of part (b): one principal's contract may not be written conditional on the agent's message to the other principal. There are several ways of defending this assumption. In a nonlinear pricing context (where two principals produce output consumed, or retailed, by a single agent), antitrust law may simply prevent principals from conditioning their pricing schedule on the quantity bought from the other principal. A similar argument can be used in a "production" example, in which a single agent produces two (separate) inputs into the principals' production processes.

What about our Coasean motivating example? Payola payments can be viewed as an indirect mechanism in which the D.J. chooses how often to play a record and gets paid according to the payola schedule. In equilibrium, there exists a one-to-one correspondence between agent types and the "decision" (how many records to play). In this context, (b) says that principals cannot condition their contracts on what is a very easily monitored observable characteristic, *viz.* a public radio program. In the Coasean example, clearly (b) is a heroic assumption.

More importantly, the published literature (Martimort (1992), Martimort (1996a)) has assumed away any direct interaction between the principals' contracts, other than through the agent's payoff function. In particular, as we will see, one principal's payoff function is usually assumed independent (other than through the contract she offers the common agent) of the other principal's contract allocation. In a non-linear pricing problem this implies, for instance, that the two principals do not compete on a product market for their output, sold through a common agent. It thus rules out most applications of economic interest, such as that of two principals selling a homogeneous (or differentiated) good through a common agent.[12] Coming back to the Coasean payola example, one record producer's profit generally depends on the other producer's sales: one should model this element of competition directly, rather than burying it in the common agent's payoff function.

Yet, concentrating on the interaction between principals' contracts purely in the agent's utility function allows us to focus on the additional source of inefficiency under common agency. Put briefly, if the decisions (in a non-linear pricing example, the quantities of the two goods) are substitutes in the agent's utility function, when one principal increases the quantity of her good exchanged with the agent (and therefore leaves the agent greater rent), the agent will wish to obtain a smaller quantity from the other principal, so that the agent's amount of rent is reduced: the principal can induce truth-telling more cheaply. The opposite holds for the case of complements.

Next, we set out the common agency model of Martimort (1992) and Stole (1992). Before solving the common agency model with non-cooperative contracting, we consider briefly the case of cooperative contracting as a benchmark for comparisons.

### 4.5.1 The Model

The structure of the model is the following. As before, let there be one agent of type $\theta \in \Theta \subset \Re$ (usually $\Theta = [\underline{\theta}, \overline{\theta}]$) where $\theta$ is only known to the agent, who may announce to principal $i$ that she is of type $\hat{\theta}_i$. Two principals $i = 1, 2$ entertain a (common) a priori belief about the agent's type given by a density function $f(\theta) > 0$ (and associated distribution function $F(\theta)$) and simultaneously and non-cooperatively choose contracts $(x_i(\hat{\theta}_i), t_i(\hat{\theta}_i))$, where $x_i : \Theta \rightarrow \Re_+$ is the agent's decision function (e.g. her consumption of principal $i$'s good in a nonlinear pricing example, or her production for principal $i$ in a production problem) and $t_i : \Theta \rightarrow \Re_+$ is a transfer payment from principal $i$ to the agent. Again, the interpretation of $t_i$ in a nonlinear pricing context is to have $t_i < 0$. Each principal $i$ has a quasi-linear utility function $v^i(x_i) - t_i$; the agent's utility function likewise is quasi-linear and given by $u(x_1, x_2, \theta) + t_1 + t_2$. Her indirect utility when she is of type $\theta$ and announces

[12]Where the literature has studied this question, the interaction is brought in through the "back door" of the agent's payoff function. Thus in Martimort (1992) or similarly in Martimort (1996a), the agent's utility function is a set of demand curves (the goods may be complements or substitutes), and principals minimize direct transfer payments to the agent but do not obtain any direct utility from the sale of their output.

$\hat{\theta}_1$, $\hat{\theta}_2$, respectively, is consequently $U(\hat{\theta}_1, \hat{\theta}_2, \theta) \equiv u(x_1(\hat{\theta}_1), x_2(\hat{\theta}_2), \theta) + t_1(\hat{\theta}_1) + t_2(\hat{\theta}_2)$. Note that we will refer to principal $i$'s rival as principal $-i$.

Before we consider the common-agency problem, in which mechanism designers choose contracts non-cooperatively, we study the benchmark case in which two principals cooperatively design mechanisms for a common agent.

### 4.5.2 The Benchmark Case

The benchmark case is that in which both principals choose contracts cooperatively. Note that in this case, the contractual relationship reduces to the bilateral principal-agent framework with a vector-valued (two dimensional) decision function. In effect, the principals offer one contract $\{x_1(\hat{\theta}), x_2(\hat{\theta}), t(\hat{\theta})\}$, and the agent's indirect utility is $U(\hat{\theta}, \theta) \equiv u(x_1(\hat{\theta}), x_2(\hat{\theta}), \theta) + t(\hat{\theta})$. Since the problem, in effect, is an application of the single-principal single-agent model, we keep our discussion brief and will only sketch proofs.

Consider first implementability (or incentive compatibility).[13] We need only consider direct revelation mechanisms, so that

$$U(\theta) \equiv U(\theta, \theta) = \max_{\hat{\theta}} u(x_1(\hat{\theta}), x_2(\hat{\theta}), \theta) + t(\hat{\theta}).$$

By the envelope theorem (for instance Milgrom (1999)), and an application of the revelation principle, we have

$$U'(\theta) = u_\theta(x_1(\theta), x_2(\theta), \theta), \tag{4.10}$$

so that the agent's rent is increasing in type. Integrating on both sides, we obtain $\int_{\underline{\theta}}^{\theta} U'(s)ds = \int_{\underline{\theta}}^{\theta} u_\theta(x_1(s), x_2(s), s)ds$, or $U(\theta) = \int_{\underline{\theta}}^{\theta} u_\theta(x_1(s), x_2(s), s)ds + U(\underline{\theta})$. However, we know (from the principals' maximization problem, and because rent is increasing in type) that $U(\underline{\theta}) = 0$

[13]Recall that a decision function $x : \Theta \to \Re_+^2$ is implementable if there exists a transfer $t(\cdot)$ such that $U(\theta, \theta) \geq U(\hat{\theta}, \theta)$, $\forall(\theta, \hat{\theta}) \in \Theta^2$.

(the lowest type agent earns no informational rent),[14] so that we finally have

$$U(\theta) = \int_{\underline{\theta}}^{\theta} u_\theta(x_1(s), x_2(s), s)ds, \tag{4.11}$$

the informational rent of a type $\theta$ agent.[15]

Equation (4.10) implies that

$$\begin{aligned}
& u_{x_1}(x_1(\theta), x_2(\theta), \theta)x_1'(\theta) + u_{x_2}(x_1(\theta), x_2(\theta), \theta)x_2'(\theta) + u_\theta(x_1(\theta), x_2(\theta), \theta) + t'(\theta) \\
= \; & u_\theta(x_1(\theta), x_2(\theta), \theta)
\end{aligned}$$

or

$$t'(\theta) = -\sum_{i=1}^{2} u_{x_i}(x_i(\theta), x_i(\theta), \theta)x_i'(\theta), \tag{4.12}$$

with the second-order condition

$$\sum_{i=1}^{2} u_{x_i\theta}(x_i(\theta), x_i(\theta), \theta)x_i'(\theta) \geq 0 \tag{4.13}$$

as in Guesnerie and Laffont (1984).

Given the Spence-Mirrlees single-crossing condition on preferences, $u_{x_i\theta} > 0$, and assuming $u_\theta > 0$, we obtain a constraint reduction theorem similar to proposition 27, above. Under the assumptions, equation (4.13) is a sufficient and necessary condition for implementability (provided, of course, that $x_i' > 0$, which may be ensured by placing well-known regularity conditions on the hazard rate $\frac{f(\theta)}{1-F(\theta)}$ and third derivatives of $u$.)

The principal's problem is to

$$\max_{x_i(\cdot), U(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \left(v^i(x_i(\theta)) + v^{-i}(x_{-i}(\theta)) + u(x_1(\theta), x_2(\theta), \theta) - U(\theta)\right) f(\theta)d\theta$$

(where the definition of $U(\theta)$ was used to replace $t(\theta)$) s.t. (4.10) and $U(\underline{\theta}) = 0$.

[14] By assumption, $du/d\theta > 0$. Therefore, rent is increasing in type. The principal minimizes costly rent by setting $U(\underline{\theta}) = 0$, so that for the lowest type agent, the participation constraint binds.

[15] The preceding discussion follows Laffont and Martimort (1997).

The Hamiltonian for this problem is

$$\begin{aligned} H(x_i, U_i, \theta, \lambda) \;&\equiv\; \left(v^i(x_i(\theta)) + v^{-i}(x_{-i}(\theta)) + u(x_1(\theta), x_2(\theta), \theta) - U(\theta)\right) f(\theta) \\ &\quad + \lambda(\theta) u_\theta(x_1(\theta), x_2(\theta), \theta). \end{aligned}$$

By Pontryagin's principle we have $\lambda'(\theta) = -\frac{\partial H}{\partial U} = f(\theta)$. Integrating from $\theta$ to $\overline{\theta}$, and making use of the transversality condition $\lambda(\overline{\theta}) = 0$, yields $\lambda(\theta) = -(1 - F(\theta))$. From $\frac{\partial H}{\partial x_i} = 0$ we obtain the familiar conditions for $i = 1, 2$

$$v^i_{x_i}(x_i(\theta)) + u_{x_i}(x_i(\theta), x_{-i}(\theta), \theta) = \frac{1 - F(\theta)}{f(\theta)} u_{x_i\theta}(x_i(\theta), x_{-i}(\theta), \theta) \qquad (4.14)$$

and $t(\theta) = \int_{\underline{\theta}}^{\theta} u_\theta(x_1(s), x_2(s), s) ds - u(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta})$.

The equilibrium in this adverse selection problem can again easily be interpreted as an equality between the marginal benefit of increasing output $f(\theta)(v^i_{x_i} + u_{x_i})$ and the marginal cost from increasing the informational rent of all types higher than $\theta$, $(1 - F(\theta))u_{x_i\theta}$.

### 4.5.3 Contracting with two Principals

The common agency model differs from the standard model in several important respects. Most obviously, there is no straightforward maximization of a single principal's objective; instead, the focus is on the (Nash) equilibrium in the mechanism design game among principals. In this game, each principal may want to design a contract such that the agent misrepresents her type to the other principal, with the aim of increasing the agent's informational rent and, therefore, the amount of rent the principal can extract. In equilibrium, of course, the agent will report truthfully to both principals, but in general this possibility will restrict the set of implementable allocations. Conditions for implementability in the common agency case will therefore differ from the conditions in the standard single-principal single-agent case (with one or multidimensional allocation functions). Relatedly, there is a new source of inefficiency in the common agency game: principals impose externalities on each other through non-cooperative contracting. We draw some classificatory distinctions below.

More subtly, in the game between contract setters, out-of-equilibrium behavior will in general influence equilibrium play. We therefore need to exercise caution before applying the revelation principle. Recall that the revelation principle states that a principal can restrict attention to direct mechanisms without loss of generality: messages sent by the agent, other than those related to type, can safely be ignored. This may no longer be true in the contracting game of common agency: messages sent by the agent to one principal may become a strategic tool for the rival principal. In general this will change the set of implementable equilibria. Before we proceed to a characterization of the implementable allocations in the common agency game, we need to address the literature on the validity of the revelation principle under common agency.

First, let us draw some distinctions to classify the nature of the externalities that arise as a consequence of non-cooperative contracting by the principals.

### 4.5.4 Classification: Externalities

We have already mentioned the externality that arises from non-cooperative contracting by several principals with a common agent, through interaction of the principals' decisions (for instance, output quantities in a non-linear pricing context) in the agent's utility function. Roughly, the mechanism is this: principal 1 will attempt to make the agent "over"-report her type to the rival principal. She does this be reducing (increasing) the "decision" $x_1$ (e.g. the quantity of output sold to the agent). When decisions are substitutes (complements) in the agent's decision function, this will lead the agent to seek an increase of the decision $x_2$ in the other principal's mechanism, by over-reporting her type. Since higher types receive greater rent, the agent retains greater informational rent, which principal 1 can then extract. So in general, we would expect the decisions (e.g. outputs) to be less than the cooperative benchmark when decisions are complements, and greater than the cooperative benchmark when decisions are substitutes.

Laffont and Martimort (1997) discuss this contractual externality in the context of organizational design: stakeholders cannot coordinate their incentive mechanisms for management but, importantly, they all have a shared objective for the firm. In other words, the only externality will be in pure rent extraction. Laffont and Martimort call this externality "type 1." We will refer to it by the more descriptive name of an indirect contractual externality (see Martimort and Stole (1999a)); "indirect" because the interaction is through the agent's utility function.

By contrast, direct contractual externalities arise through the "direct" route of the principals' objectives. If principal 1 cares not only about her own allocation, but also about principal 2's allocation, this will of course lead to externalities in the usual way. Laffont and Martimort (1997) call this externality "type 2." Again following the usage in Martimort and Stole (1999a), we will refer to this type of externality as a direct contractual externality. We have already drawn attention to the fact that most of the common agency literature has ignored these direct externalities. We follow this pattern in our exposition of the theory, although we will return to the issue after our discussion of the "standard" common agency game.

### 4.5.5 The Revelation Principle under Common Agency

Before we can characterize implementability in the common agency context, we need to comment on the applicability of the revelation principle. Although above we have already set up the model restricted to direct revelation mechanisms, it turns out that we need to exercise caution before applying the revelation principle to situations of common agency.

The centralized version of the revelation principle of section 4.2 remains true, virtually by definition, in situations of common agency, where each agent is given mechanisms from several mechanism designers.

For instance, let the number of mechanism designers, indexed by $i$, be two. The definitions made in section 4.2 are extended easily to this case (which they, in fact, imply). We use the notation introduced in section 4.2 with only minimal alterations to accommodate the two-principal case. Let a social choice rule be the pair of functions $g_i : \Theta \rightarrow X_i$, $i = 1, 2$, and denote a pair of general mechanisms by the game form $\Gamma_S \equiv (S_{11}, ..., S_{1j}, h_1, S_{21}, ..., S_{2j}, h_2)$, where the additional subscripts now refer to the mechanism given by mechanism designer $i = 1, 2$. Extend the definition of a dominant strategy equilibrium to the pair of strategy profiles $(s_1^*, s_2^*)$, and define implementability of $(g_1, g_2)$ in dominant strategies by $\Gamma_G$ as the existence of a dominant strategy equilibrium $(s_1^*, s_2^*)$ such that $h_i(s_i^*(\theta)) \equiv g_i(\theta)$, for $i = 1, 2$. Finally, let $(g_1, g_2)$ be truthfully implementable in dominant strategies if, for all agents $j$, and all designers $i$, $s_{ij}^*(\theta_j) \equiv \theta_j$, and $(s_1^*, s_2^*)$ is a dominant strategy equilibrium of the direct mechanism $\Gamma_\Theta \equiv (\Theta_1, ..., \Theta_j, g_1 \Theta_1, ..., \Theta_j, g_2)$. Then we have the following simple special case:

**Corollary 28** *If in a situation where agents are given mechanisms from two mechanism designers there exists some general mechanism $\Gamma_G$ that implements $(g_1, g_2)$ in dominant strategies, then $(g_1, g_2)$ is also truthfully implementable in dominant strategies.*

**Proof.** The proof follows from the definitions and mirrors precisely the proof in the general case. ■

(Note that the corollary covers the case in which the allocation rules $(h_1, h_2)$ in the mechanisms $\Gamma_S$ are restricted to being equilibria in the game among mechanism designers. However, no such restriction can be placed on $(g_1, g_2)$.)

The interpretation of this last result is that each agent represents her type truthfully as $\theta_j$ to an independent mediator, and the outcome is calculated from the pair of functions $(g_1, g_2)$, where $g_i \equiv h_i \circ s_i^*$. Since $s_{ij}^*$ is the equilibrium strategy agent $j$ would use in designer $i$'s mechanism under the rule $h_i$, there are no gains to misrepresentation if $g_i$ is used.

Although trivially true, the standard (centralized) version of the revelation principle in corollary 28 may not hold much interest in the common agency case, unless institutional features guarantee the existence of an independent mediator, as required by the centralized version of the revelation principle. The question then arises whether a decentralized version of the revelation principle under common agency also holds.

In the discussion that follows (and for the remainder of the paper), two principals design contracts for a single, common agent. To simplify notation, we can therefore write $s_i$ instead of $s_{ij}$.

Making use of the definitions introduced above, a decentralized version of the revelation principle for the case of two mechanism designers and a single agent, could be the following:

**Conjecture 29** *If there exists some general mechanism $\Gamma_S \equiv (S_1, h_1, S_2, h_2)$ that implements some choice rule $(g_1, g_2)$ in dominant strategies and for which $(h_1, h_2)$ is a Nash equilibrium in the game among mechanism designers, then $(g_1, g_2)$ can also be implemented through the direct mechanism $\Gamma_\Theta \equiv (\Theta, g_1, \Theta, g_2)$ in which the (dominant strategy) equilibrium strategies are $s_i^* = \theta$ for $i = 1, 2$ (i.e. $(g_1, g_2)$ is truthfully implementable in dominant strategies), and for which $(g_1, g_2)$ is a Nash equilibrium in the game among mechanism designers.*

Unfortunately, as Martimort and Stole (1997) and Martimort and Stole (1999b) point out, this conjecture, in general, is false. They show the failure of this conjectured revelation principle in common agency situations by counterexample, which we reproduce here.

Suppose that the agent's type is known (so that adverse selection issues may be conveniently ignored). Consider the following game between two principals, 1 and 2. Payoffs for principal 1, principal 2 and the agent are given as ordered triples:

| | $x_2 = A$ | $x_2 = B$ | $x_2 = C$ |
|---|---|---|---|
| $x_1 = A$ | $(1,1,1)$ | $(2,0,2)$ | $(-1,5,10)$ |
| $x_1 = B$ | $(0,2,2)$ | $(1,1,1)$ | $(0,0,0)$ |
| $x_1 = C$ | $(5,-1,10)$ | $(0,0,0)$ | $(0,0,0)$ |

In a general, indirect mechanism, agents can send messages from some exogenously determined message space. Each principal $i$ offers an allocation $x_i$ to the agent, as a function of the agent's message. Suppose the message space is exogenously restricted to be $S_i = \{s_i', s_i''\}$. Let each principal's contract offer be: $x_i = B$ if the agent sends message $s_i'$ and $x_i = C$ if the agent sends message $s_i''$. This of course is a Nash equilibrium in the game among principals. The agent will report $s_i'$ to both principals and the outcome of this indirect mechanism contracting game is $(B, B)$. An alternative contract offer by the principals could be: $x_i = C$ whatever message the agent sends. This is again a Nash equilibrium in the game among principals, and it implements the outcome $(C, C)$. So there exists an indirect mechanism that implements at least two outcomes: $(B, B)$ and $(C, C)$.

In a direct mechanism, agents can only send messages about their type, and principals choose contract offers as functions of the agent's message. In our context, since the agent's type space is degenerate (i.e. the agent's type is known), principals are restricted to choosing a single allocation. The only Nash equilibrium in the above game with such direct mechanisms is $(C, C)$.

This example illustrates simply that restriction of mechanisms to direct mechanisms is not without loss of generality: equilibria that are implementable in the indirect mechanism (with sufficiently rich message space), are not implementable in the direct mechanism. The reason for this is that off-equilibrium contract actions sustain an equilibrium in the indirect mechanism that cannot be sustained in a direct mechanism. Take, for instance, the Nash equilibrium in the indirect mechanism, in which each principal offers the contract: $x_i = B$ if the agent sends message $s_i'$ and $x_i = C$ if the agent sends message $s_i''$. Offering $C$ (although in equilibrium this will not be implemented, given the contracts), sustains this equilibrium. If principal 1, for instance, only offered $x_1 = B$ whatever the agent's message, principal 2 would deviate to offering only $x_2 = A$ whatever the agent's message.

This suggests a nestedness of implementable equilibria in direct and indirect mechanisms, but in fact the sets of equilibria are non-nested: there also exist equilibria in the direct mechanism (consider the mixed strategies $(\frac{1}{12}, \frac{5}{12}, \frac{1}{2})$ for both principals; then if one principal deviates to the contract of the above form "$x_i = B$ if the agent sends message $s_i'$ and $x_i = C$ if the agent sends message $s_i''$" her payoff is strictly improved). In general the step from considering indirect mechanisms (with given message spaces) to considering only direct mechanisms (where the message space is the agent's type space) is therefore with loss of generality.

Martimort and Stole (1999b) therefore propose a change in focus: rather than concentrating on direct mechanisms, we should study economically more meaningful indirect mechanisms, such as non-linear pricing schedules, for instance. Martimort and Stole (1999b) prove an extension of the taxation principle studied in section 4.4.3. We discuss their results below. First, we have a different suggestion for the recovery of some of the flavor of the revelation principle in common agency situations.

We propose that for many applications, a simpler version of the revelation principle may be sufficient. Often, the nature of the problem naturally restricts principals to the use of direct mechanisms. For instance, an insurer may ask the financial adviser for age, sex, medical history, of a prospective insured; a firm may ask its retailer for specifics about market conditions, a regulator may ask the regulated firm to report on its cost; etc. Although the agent may seek to misrepresent the required information, in many economically meaningful contexts, principals seek information about a specific characteristic (type). That is, the agent's message space is exogenously imposed to be the same as her type space. In this context, we may ask the question of whether restriction to mechanisms in which the agent reports truthfully on her type is without loss of generality. A revelation principle for this case can easily be proven.

Consider a situation in which two principals non-cooperatively design contracts for a single agent. We modify the definitions made in 4.2 for this case:

**Definition 30** *A pair of* choice functions $(g_1, g_2)$ *is a pair of functions* $g_i : \Theta \rightarrow X_i$, $i = 1, 2$.

Let $s_i \in S$ be the agent's strategy in principal $i$'s mechanism, and define a pair of outcome functions $(h_1, h_2)$, where $h_i : S \rightarrow X_i$.

**Definition 31** *A pair of outcome functions* $(h_1, h_2)$ *is a Nash equilibrium if, for all* $h_i'(\cdot)$,

$$\int_{\underline{\theta}}^{\overline{\theta}} v^i(h_i^*(\hat{\theta}_i^*))dF(\theta) \geq \int_{\underline{\theta}}^{\overline{\theta}} v^i(h_i'(\hat{\theta}_i^*))dF(\theta).$$

*(Note that* $\hat{\theta}_i^*$ *of course depends on the mechanism offered by principal* $-i$, $h_{-i}^*$.*)*

**Definition 32** *A pair of general* mechanisms *with the exogenously determined strategy space* $S$ *is a game form* $\Gamma_S \equiv (S, h_1, S, h_2)$.

**Definition 33** *The strategies* $(s_1^*, s_2^*)$ *are a* dominant strategy equilibrium *of* $\Gamma_S$ *if, for all* $s_i' \in S$:

$$u(h_1(s_1^*), h_2(s_2^*), \theta) \geq u(h_1(s_1'), h_2(s_2'), \theta).$$

**Definition 34** *The pair of social choice functions* $(g_1, g_2)$ *is* implementable in dominant strategies by $\Gamma_S$ *if there exists a dominant strategy equilibrium* $(s_1^*, s_2^*)$ *of* $\Gamma_S$ *such that* $h_i(s_i^*(\theta)) = g_i(\theta)$ *for all* $\theta$, *and* $i = 1, 2$.

**Definition 35** *A pair of* direct mechanisms *is a game form* $\Gamma_\Theta \equiv (\Theta, \cdot, \Theta, \cdot)$, *i.e. a pair of general mechanisms in which* $S = \Theta$.

**Definition 36** *The pair of social choice functions* $(g_1, g_2)$ *is* truthfully implementable in dominant strategies *if, for all* $i$, $s_i^*(\theta) = \theta$ *and* $s^*$ *is a dominant strategy equilibrium of the pair of direct incentive-compatible (or, direct revelation) mechanisms* $\Gamma_\Theta \equiv (\Theta, g_1, \Theta, g_2)$.

A version of the revelation principle under common agency that is sufficient when we can exogenously restrict the agent's message space to messages on her type, is the following:

**Proposition 37** *If there exists some pair of direct mechanisms $\Gamma_\Theta \equiv (\Theta, h_1, \Theta, h_2)$ that implements some pair of choice rules $(g_1, g_2)$ in dominant strategies and for which $(h_1, h_2)$ is a Nash equilibrium in the game among mechanism designers, then $(g_1, g_2)$ can also be implemented through the pair of direct mechanisms $\tilde{\Gamma}_\Theta \equiv (\Theta, g_1, \Theta, g_2)$ in which the dominant strategy equilibrium strategies are $s_i^* = \theta$ (i.e. $(g_1, g_2)$ is truthfully implementable in dominant strategies), and for which $(g_1, g_2)$ is a Nash equilibrium in the game among mechanism designers.*

**Proof.** Suppose $\Gamma_\Theta \equiv (\Theta, h_1, \Theta, h_2)$ implements $(g_1, g_2)$ in dominant strategies using the strategy profile $s^* = (\hat{\theta}_1^*, \hat{\theta}_2^*)$, and that the pair of allocation functions $(h_1, h_2)$ are a Nash equilibrium in the game between mechanism designers. Then, by the definition of a dominant strategy equilibrium, for all $\theta$, $\hat{\theta}_1'$, $\hat{\theta}_2'$,

$$u(h_1^*(\hat{\theta}_1^*(\theta|h_2^*)), h_2^*(\hat{\theta}_2^*(\theta|h_1^*)), \theta) \geq u(h_1^*(\hat{\theta}_1'), h_2^*(\hat{\theta}_2'), \theta).$$

In particular, we have

$$u(h_1^*(\hat{\theta}_1^*(\theta|h_2^*)), h_2^*(\hat{\theta}_2^*(\theta|h_1^*)), \theta) \geq u(h_1^*(\hat{\theta}_1^*(\theta'|h_2^*)), h_2^*(\hat{\theta}_2^*(\theta''|h_1^*)), \theta).$$

From the Nash equilibrium property we know that, for all $i$, and all $h_i'$,

$$\int_{\underline{\theta}}^{\overline{\theta}} v^i(h_i^*(\hat{\theta}_i^*(\theta|h_{-i}^*)))dF(\theta) \geq \int_{\underline{\theta}}^{\overline{\theta}} v^i(h_i'(\hat{\theta}_i^*(\theta|h_{-i}^*)))dF(\theta).$$

By implementation, $g_i(\theta) \equiv h_i^*(\hat{\theta}_i^*(\theta|h_{-i}^*))$, so we have

$$u(g_1(\theta), g_2(\theta), \theta) \geq u(g_1(\theta'), g_2(\theta''), \theta),$$

and, letting $g_i' \equiv h_i' \circ \hat{\theta}_i^*$, we obtain, for all $g_i'$,

$$\int_{\underline{\theta}}^{\overline{\theta}} v^i(g_i(\theta|h_{-i}^*))dF(\theta) \geq \int_{\underline{\theta}}^{\overline{\theta}} v^i(g_i'(\theta|h_{-i}^*))dF(\theta),$$

which proves the proposition. ∎

This admittedly stripped-down version of the revelation principle says that if we can restrict attention to direct mechanisms, we can model the mechanism design game without loss of generality so that the agent reports her type truthfully. For many economically significant settings, this version of the revelation principle may be sufficient.

Our result suggests (somewhat unsurprisingly) that the failure of the revelation principle stems from the step from indirect to direct mechanisms, not from the requirement for truthful reporting. In this sense, it reinforces a point made in the simple (degenerate type space) example of Martimort and Stole (1999b). Recall that in their example, the failure of the revelation principle occurred without informational asymmetry. Our result reinforces this point in locating the failure of the revelation principle away from the (informational) requirement for truthful reporting.

A different route is taken by Epstein and Peters (1999). They suggest the following intuition: the agent's private information with respect to principal 1, for instance, includes not only the agent's type, but also the contract offered to her by agent 2. If a direct mechanism takes this fuller definition of "type" into account, the revelation principle should hold as usual. The problem then is to show that the agent's type is part of a space that converges: since the agent's type includes information on one principal's contract, which depends on the other principal's contract, etc., this is not automatic. Epstein and Peters prove that, indeed, the agent's type space converges to a "universal type space"—and in this context, the revelation principle holds. However, these universal type spaces may be hard to characterize. We therefore turn briefly to the somewhat simpler, but economically intuitive solution offered by Martimort and Stole in recent unpublished work.

### 4.5.6 A Taxation Principle for Common Agency Games

In two recent papers (Martimort and Stole (1999b) and Martimort and Stole (1999a)), Martimort and Stole suggest a change of focus towards indirect mechanisms (such as non-linear pricing schedules) which have a certain economic appeal. They prove an extension of the taxation principle (cf. section 4.4.3, above) for common agency games: instead of maximizing over all indirect mechanisms, the principals can, without loss of generality, restrict attention to non-linear pricing schedules.[16]

[16] We discuss here a simple version of the "extended taxation principle," for economic situations where an agent's allocation consists of a decision (e.g. quantity) and a transfer (e.g. price). This is the version

The intuition for this result is simple. Recall that the agent's payoff is quasilinear in transfers, $u(x_1, x_2, \theta) + t_1 + t_2$. Consider a non-linear pricing game, so that the agent's payoff is $u(x_1, x_2, \theta) + \tilde{t}_1(x_1) + \tilde{t}_2(x_2)$, and an "augmented" non-linear pricing game (in which the agent can additionally send messages $m_1 \in M_1$, and $m_2 \in M_2$, so that her payoff is $u(x_1, x_2, \theta) + t_1(x_1, m_1) + t_2(x_2, m_2)$. Recall that in a non-linear pricing context, the $t_i$ are negative, so that payments flow from the agent to the principals. Consider the "augmented" non-linear pricing game. In this case, for given contracts, the agent chooses $x_i$ and $m_i$ $(i = 1, 2)$ so as to maximize her utility; since utility is quasilinear in transfers, this is the same as minimizing transfers (recall $t_i$ are negative) with respect to $m_i$, and then maximizing with respect to $x_i$. The principals need therefore only pay attention to the lower envelope of the possible $t_i(\cdot, \cdot)$ functions. Defining $\tilde{t}_i(\cdot) = \min_{m_i} t(\cdot, m_i)$, Martimort and Stole (1999a) can easily show that any Nash equilibrium in augmented non-linear pricing schedules can be attained using a simple non-linear pricing schedule.[17]

The modelling strategy is therefore the following: find a Nash equilibrium in non-linear pricing schedules, using the revelation principle to check whether indeed each principal's strategy is a best response to the other principal's strategy. In this way, all equilibria of any mechanism with general message spaces can be uncovered. (Note that the use of the revelation principle to find one principal's best response to a given mechanism from the other principal is without loss of generality: the problem is the same as in the standard single-principal single-agent model. The difficulty in applying the revelation mechanism in common agency games arises when we attempt to use the revelation principle to characterize the equilibria in the mechanism design game among principals.)

Non-linear pricing schedules are often much simpler to characterize than general indirect mechanisms with general message spaces. Apart from their direct economic appeal they also possess nice theoretical properties in common agency games.

The revelation principle in effect ties the principal to offering each agent just one allocation (the equilibrium allocation for that type). The difficulty in using the revelation principle in common agency games is that off-equilibrium offers can sustain equilibria. Therefore, if

in Martimort and Stole (1999a). A more general version of the principle is proven in Martimort and Stole (1999b). As before, we also restrict attention to deterministic contracts and pure strategies.

[17]In a related paper, Martimort and Stole (1999b) prove a more general version of the "extended taxation principle" for non-quasilinear utility and possibly mixed strategies.

we are to use the revelation principle (as we will), one must be careful to extend the contract offers beyond the equilibrium allocation (in the manner of a non-linear tariff, which presents a complete menu of options), to capture all equilibria arising in indirect mechanism setting games.

### 4.5.7 The Common Agency Model

We now turn to a characterization of implementable and feasible allocations in common agency games with two principals $i = 1, 2$, who simultaneously and non-cooperatively offer mechanisms to a single common agent. We follow the route taken in the single-principal single-agent case and characterize implementability before proceeding to a description of the optimal contract offers for the two principals. This section follows the models proposed independently by Martimort (1992) and Stole (1992).

In analogy to the single-principal single-agent case, define common implementability as follows:

**Definition 38** *An allocation* $(x_1(\cdot), x_2(\cdot), t_1(\cdot), t_2(\cdot))$ *is commonly implementable if there exist transfer schedules* $t_1(\cdot), t_2(\cdot)$ *such that the agent's incentive compatibility constraint is satisfied, that is, if for all* $(\theta, \hat{\theta}_1, \hat{\theta}_2) \in \Theta^3$

$$u(x_1(\theta), x_2(\theta), \theta) + t_1(\theta) + t_2(\theta) \geq u(x(\hat{\theta}_1), x_2(\hat{\theta}_2), \theta) + t_1(\hat{\theta}_1) + t_2(\hat{\theta}_2),$$

*or, in terms of indirect utility, for all* $\theta \in \Theta$,

$$U(\theta, \theta, \theta) = \max_{\hat{\theta}_1, \hat{\theta}_2 \in \Theta^2} U(\hat{\theta}_1, \hat{\theta}_2, \theta).$$

**Definition 39** *An allocation* $(x_1(\cdot), x_2(\cdot), t_1(\cdot), t_2(\cdot))$ *is commonly feasible if* $U(\theta, \theta, \theta) \geq 0$ $\forall \theta \in \Theta$.

Consider first incentive compatibility. Restricting attention to direct mechanisms, the agent's best reply functions are defined as $\hat{\theta}_1(\hat{\theta}_2, \theta | x_1, x_2, t_1, t_2) = \arg\max_{\tilde{\theta}} U(\tilde{\theta}, \hat{\theta}_2, \theta)$ and $\hat{\theta}_2(\hat{\theta}_1, \theta | x_1, x_2, t_1, t_2) = \arg\max_{\tilde{\theta}} U(\hat{\theta}_1, \tilde{\theta}, \theta)$. In equilibrium, of course, $\hat{\theta}_1 = \hat{\theta}_2 = \theta$. Necessary conditions for implementability can be derived as in the following:[18]

[18] cf. Stole (1992)

**Proposition 40 (Stole)** *An allocation is commonly implementable only if:*

$$U_{\hat{\theta}_i}(\theta,\theta,\theta) = 0,\ i = 1,2$$

$$U_{\hat{\theta}_i\theta}(\theta,\theta,\theta) + U_{\hat{\theta}_1\hat{\theta}_2}(\theta,\theta,\theta) \geq 0$$

$$U_{\hat{\theta}_1\theta}(\theta,\theta,\theta)U_{\hat{\theta}_2\theta}(\theta,\theta,\theta) + U_{\hat{\theta}_1\hat{\theta}_2}(\theta,\theta,\theta)\left(U_{\hat{\theta}_1\theta}(\theta,\theta,\theta) + U_{\hat{\theta}_2\theta}(\theta,\theta,\theta)\right) \geq 0$$

*or, analogously:*

$$t_i' = -u_{x_i}(x_1,x_2,\theta)x_i',\ i = 1,2 \tag{4.15}$$

$$u_{x_1x_2}(x_1,x_2,\theta)x_1'x_2' + u_{x_i\theta}(x_1,x_2,\theta)x_i' \geq 0,\ i = 1,2 \tag{4.16}$$

$$u_{x_1\theta}u_{x_2\theta}x_1'x_2' + u_{x_1x_2}x_1'x_2'[u_{x_1\theta}x_1' + u_{x_2\theta}x_2'] \geq 0 \tag{4.17}$$

Martimort (1992) obtains identical conditions in his equations (12) and (13). The proof follows along the lines of the single-principal single-agent case and is therefore omitted. What becomes clear from the second-order conditions is that, in general, in the multiprincipal setting, the solution to the contracting problem will depend on whether $x_1$ and $x_2$ are contract substitutes ($u_{x_1x_2} < 0$) or contract complements ($u_{x_1x_2} > 0$).

If $u_{x_1x_2} = 0$, the problem reduces to two unrelated standard single-principal single-agent problems, so that a version of the constraint reduction theorem (proposition 27) applies. In general, however, the necessary conditions (4.15)–(4.17) are not sufficient. A sufficient condition for implementability is (cf. Stole (1992, Theorem 5)) that, for all $(\hat{\theta}_1,\hat{\theta}_2,\theta) \in \Theta^3$,

$$\int_\theta^{\hat{\theta}_2}\int_\theta^{\hat{\theta}_1} U_{\hat{\theta}_1\hat{\theta}_2}(t,s,\theta)dtds + \int_\theta^{\hat{\theta}_2}\int_s^{\theta}\left(U_{\hat{\theta}_1\hat{\theta}_2}(t,s,t) + U_{\hat{\theta}_2\theta}(t,s,t)\right)dtds +$$

$$+\int_{\theta}^{\hat{\theta}_1}\int_{s}^{\theta}\Big(U_{\hat{\theta}_1\hat{\theta}_2}(s,t,t)+U_{\hat{\theta}_1\theta}(s,t,t)\Big)\,dtds \le 0$$

and $U_{\hat{\theta}_i}(\theta,\theta,\theta)=0$. The proof is simple and is therefore not reproduced here. For details cf. Stole (1992).

To see why sufficiency in the common agency case is a more complex problem, consider the case where $u_{x_1x_2}=0$, so that (4.16) and (4.17) reduce to $u_{x_i\theta}x_i' \ge 0$. This is just the condition for implementability in the single-principal single-agent case, so that $x_i' \ge 0$ is sufficient for implementability. (Compare this reduced second order condition with (4.13).) Now suppose $u_{x_1x_2}\neq 0$, but that the agent's report to principal $-i$ is known to be $\hat{\theta}_{-i}=\theta$. Then (4.16) and (4.17) both reduce to $u_{x_i\theta}x_i'+u_{x_1x_2}x_{-i}' \ge 0$. But this last condition cannot be sufficient for implementability: additional constraints are needed to ensure that $\hat{\theta}_{-i}=\theta$ for any contract $\{x_i,t_i\}$, hence the complex sufficiency condition above. In general, principal $i$'s contract choice will have an external effect on the agent's report $\hat{\theta}_{-i}$.

Finally note that for independence of $x_1$ and $x_2$ in the agent's utility function, the integral condition, above, is sufficient for the conditions

$$u(x_1(\theta),x_2(\theta),\theta)+t_1(\theta)+t_2(\theta)\ge u(x_1(\hat{\theta}_1),x_2(\theta),\theta)+t_1(\hat{\theta}_1)+t_2(\theta),$$

$$u(x_1(\theta),x_2(\theta),\theta)+t_1(\theta)+t_2(\theta)\ge u(x_1(\theta),x_2(\hat{\theta}_2),\theta)+t_1(\theta)+t_2(\hat{\theta}_2),$$

which, of course, are just the implementability conditions for two separate contracts for the decisions $x_1$ and $x_2$.

Now that implementability is characterized, we study the principals' optimal contract choices. Again we follow Martimort (1992) and Stole (1992).

Consider principal $i$'s contract choice. Recall that equation (4.11), which states that $U(\theta)=\int_{\underline{\theta}}^{\theta}u_\theta(x_1(s),x_2(s),s)ds$, is the informational rent extracted by an agent of type $\theta$. Provided the Spence-Mirrlees condition holds ($u_{x_i\theta}>0$), a decrease in $x_i$ reduces the agent's informational rent. When the decisions are substitutes, a decrease in decision $x_i$ will lead to an increase in $x_{-i}$, which tends to increase the agent's informational rent. Principal $i$'s incentive to reduce $x_i$ through her contract choice is therefore reduced, and in general the decision will be greater than that under cooperation. In the case of contract complements, a decrease in $x_i$ in order to reduce the agent's rent will lead the agent to reduce $x_{-i}$ also, so that her rent is reduced further. Generally, the agent's decision will be lower than that in the benchmark case. This illustrates the nature of the indirect contractual (type 1) externality, an externality in pure rent extraction.

We can be more specific about this externality. Recall the definition of the agent's best response functions, $\hat{\theta}_1$ and $\hat{\theta}_2$, and consider how this report changes in response to a change in the principals' contracts. Stole (1992, Theorem 8) quantifies the externality of principal $i$'s contract choice on the agent's report to principal $-i$:

$$\frac{\partial \hat{\theta}_{-i}[\theta|x_i]}{\partial x_i} = \begin{cases} u_{x_1 x_2}(x_1, x_2, \theta)/[u_{x_{-i}\theta}(x_i, x_{-i}, \theta) + u_{x_1 x_2}(x_i, x_{-i}, \theta)x_i'] & \text{if } x_{-i}' > 0 \\ 0 & \text{if } x_{-i}' = 0 \end{cases} .$$

It is clear that, when decisions are complements ($u_{x_1x_2} > 0$), the agent's report to principal $-i$ will increase as her decision under principal $i$'s contract increases. When the decisions are substitutes ($u_{x_1x_2} < 0$), an increase of the decision under principal $i$'s contract will tend to reduce the agent's report to principal $-i$.[19]

To solve the model, consider the agent's rent

$$U(\theta) = \max_{\hat{\theta}_1, \hat{\theta}_2} u(x_1(\hat{\theta}_1(\theta)), x_2(\hat{\theta}_2(\theta)), \theta) + t_1(\hat{\theta}_1(\theta)) + t_2(\hat{\theta}_2(\theta)).$$

From the envelope theorem we obtain $U'(\theta) = u_\theta(x_1(\hat{\theta}_1(\theta)), x_2(\hat{\theta}_2(\theta)), \theta)$. By the revelation principle (which here is admissible: we seek to define one principal's best report correspondence to the other principal's mechanism), we have $\hat{\theta}_1(\theta) = \theta$, so that we obtain

$$U'(\theta) = u_\theta(x_1(\theta), x_2(\hat{\theta}_2(\theta)), \theta). \tag{4.18}$$

Compare this with the expression for the change in the agent's rent with respect to type in the standard model, equation (4.5). Again, the agent's rent increases in type.

[19]Stole (1992) obtains sufficient conditions under contract complements, for the second-order conditions (4.16) and (4.17) to be sufficient as well as necessary for implementability. These conditions imply also that $u_{x_1x_2}x_i' + u_{x_{-i}\theta} \geq 0$, so that the statement about the agent's report under complements can be made precise.

The principal's problem is to

$$\max_{x_1(\cdot),\hat{\theta}_2(\cdot),U(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \left(v^1(x_1(\theta)) + u(x_1(\theta), x_2(\hat{\theta}(\theta)), \theta) + t_2(\hat{\theta}_2(\theta)) - U(\theta)\right) f(\theta) d\theta$$

s.t. (4.18), $U(\underline{\theta}) = 0$ (assuming that $U'(\theta) > 0$), and

$$u_{x_2}(x_1(\theta), x_2(\hat{\theta}_2(\theta)), \theta) x_2'(\hat{\theta}_2(\theta)) + t_2'(\hat{\theta}_2(\theta)) = 0$$

($\hat{\theta}_2(\cdot)$ is chosen optimally).

The Hamiltonian for this problem is

$$\begin{aligned} H(x_1, U, \theta, \lambda) \equiv & \left(v^1(x_1(\theta)) + u(x_1(\theta), x_2(\hat{\theta}_2(\theta)), \theta) + t_2(\hat{\theta}_2(\theta)) - U(\theta)\right) f(\theta) \\ & + \lambda(\theta) \left(u_\theta(x_1(\theta), x_2(\hat{\theta}_2(\theta)), \theta)\right) \\ & - \mu \left(u_{x_2}(x_1(\theta), x_2(\hat{\theta}_2(\theta)), \theta) x_2'(\hat{\theta}_2(\theta)) + t_2'(\hat{\theta}_2(\theta))\right) d\theta \end{aligned}$$

By Pontryagin's principle we have $-\frac{\partial H}{\partial U} = \lambda'(\theta)$, so that $\lambda(\theta) = -(1 - F(\theta))$; $\frac{\partial H}{\partial x_1} = 0$, so that $v^1_{x_1} + u_{x_1} = \frac{1-F(\theta)}{f(\theta)} u_{x_1\theta} + \mu \frac{1}{f(\theta)} u_{x_1 x_2 x_2'}$, and $\frac{\partial H}{\partial \hat{\theta}_2} = 0$, so that (simplifying by differentiating (4.15) with respect to $\theta$ and making use of the fact that $u_{x_2} x_x' + t_2'$ vanishes) $(1 - F(\theta)) u_{x_2\theta} x_2' - \mu(u_{x_1 x_2} x_1' x_2' + u_{x_2\theta} x_2') = 0$. This yields the following characterization of principal $i$'s best response to principal $-i$'s contract offer; given sufficient care is taken in extending the best response function appropriately, it also characterizes an equilibrium $\{x_1, x_2\}$ with increasing decision functions in the common agency game:

$$v^i_{x_i} + u_{x_i} = \frac{1 - F(\theta)}{f(\theta)} \left[u_{x_i\theta} + \frac{u_{x_{-i}\theta} u_{x_1 x_2} x'_{-i}}{u_{x_{-i}\theta} + u_{x_1 x_2} x_i'}\right] \quad (4.19)$$

with

$$t(\theta) = \int_{\underline{\theta}}^{\theta} u_{x_i}(x_1(s), x_2(s), s) x_i'(s) ds + \alpha_i u(x_1(\underline{\theta}), x_2(\underline{\theta}), \underline{\theta})$$

for some $\alpha_i$ such that $\alpha_1 + \alpha_2 = 1$ (Stole (1992, Theorems 9, 11) and Martimort (1992)). Note that, as before, there is no distortion in the decisions for the "best" type $\overline{\theta}$ agent. Note also that there is now an additional distortionary term $\frac{u_{x_{-i}\theta} u_{x_1 x_2} x'_{-i}}{u_{x_{-i}\theta} + u_{x_1 x_2} x_i'}$, which is positive for contract complements, and negative for contract substitutes (if the second-order conditions are necessary and sufficient for implementability, i.e. if $u_{x_{-i}\theta} + u_{x_1x_2}x_i' \geq 0$). From the concavity of $v(\cdot)$ and $u(\cdot)$ we can therefore heuristically conclude that when decisions are complements, decisions will be lower than in the cooperative benchmark case, and when they are substitutes, they should, in general, be higher.

Generally, we would now confirm that the $x_i$ are nondecreasing. Under common agency, this is much harder than in the standard single-principal single-agent model. When decisions are complements, restrictions on some third derivatives ($u_{x_ix_x\theta} + u_{x_1x_2\theta} \leq 0$, $u_{x_1x_2\theta} \leq 0$, $u_{x_i\theta\theta} \leq 0$) give the desired conclusion on $x_i$. In this case, also, there exists a continuum of equilibria; all have the property that the equilibrium decision (as anticipated earlier) is lower than that in the cooperative benchmark case.

When decisions are substitutes, proving that the $x_i$ are nondecreasing is even more difficult. One sufficient condition is linearity of the inverse hazard rate, quadratic utility, and the somewhat hard to interpret condition $\frac{v^i_{x_ix_i}+u_{x_ix_i}}{u_{x_1x_2}} \geq (1+\gamma)\frac{u_{x_i\theta}}{u_{x_j\theta}}$. In this case also, uniqueness of equilibrium can be established.[20]

### 4.5.8 The Way Ahead

A study of the pure rent-extraction externalities between principals in a common agency game, while a useful and, as we have seen, insightful exercise, does not yet answer one question that our (Coasean) organizing example raises: How are common agency models applicable to situations in which principals compete against each other, not just in rent extraction from a common agent, but also, through the agent, on a common product market? Many common agency problems of real interest are of this type: How do utilities price products (electricity or gas supply, telephone service) when different suppliers compete for the same customer? How do airlines sell tickets to agents when an agent could choose amongst any number of (more-or-less homogenous) carriers? How do insurers with similar or identical products compete via a common sales agent (independent financial adviser)?

In all these examples, there is an additional conduit for an externality between principals: if the agent buys more of one principal's good, she will buy less of the other's. In general, we would want to include the entire vector of decisions in each principal's utility function. For

[20] cf. Stole (1992)

instance, in a "typical" non-linear pricing game (e.g. two electricity suppliers competing for a single customer), if both were to offer strictly concave non-linear tariffs, one should expect a boundary solution: the agent would presumably buy the homogenous good from only one supplier. The principal who currently sells none of her product to the agent might then consider offering a less concave pricing schedule to attract some purchases from the agent, thus compromising informational rent-extraction.

Much of this is still unclear. In recent unpublished work, Martimort and Stole (1999a) have begun to address this class of problems.

### The Common Agency Model with Direct Contractual Externalities

Martimort and Stole (1999a) model a situation in which two principals $i = 1, 2$ buy intermediate goods $x_1, x_2$ from a common agent, and produce final outputs that are perfect substitutes on the product market. Principals' production is costless and such that from every unit of input, one unit of output is produced. The market for final goods is characterized by an inverse demand curve $p(x_1+x_2)$. We therefore interpret the agent's utility $u(x_1, x_2, \theta)$ as the agent's production cost of the two intermediate goods and accordingly as nonpositive. $\theta$ is the agent's efficiency parameter. Transfers flow from principals to agent, so the $t_i$ are positive. Principal $i$'s payoff is the profit from selling $x_i$, that is $v^i(x_i, x_{-i}) = p(x_1 + x_2)x_i$. Therefore the only change from our earlier model is that $x_{-i}$ enters in principal $i$'s payoff function. Nor is the methodological approach to solving for the equilibrium contracts altered: Each principal takes the other principal's contract (here, non-linear pricing schedule) as given, and calculates (using the revelation principle) her own best response. In this exercise, all the results from the standard principal-agent model (and in particular, the constraint reduction theorem) can be used.[21]

For bounded support of the agent's information parameter $\theta$, there exist a multiplicity of Nash equilibria, each supported by different conjectures about off-equilibrium path behavior. When intermediate goods are substitutes in the agent's utility function, total (final) output will be between the Cournot and the Bertrand quantities; when intermediate goods are complements, total output will lie between the monopoly and the Cournot quantities. In

[21] The exception is that the single-crossing property $u_{x_i\theta} > 0$ cannot automatically be assumed, since $u$ will in general depend on principal $-i$'s mechanism. Martimort and Stole (1999a) therefore restrict attention to equilibria in which this condition holds.

the spirit of Klemperer and Meyer (1989), Martimort and Stole introduce more uncertainty into the model (by giving the agent's private information parameter unbounded support), and are then able to select a unique equilibrium. Interestingly, the equilibrium output lies strictly between the Cournot and Bertrand outputs for the lowest type agent, when intermediate goods are either substitutes or complements. This game therefore implements neither the most collusive, nor the most competitive outcome for the principals.

### Comment

While the literature has moved towards addressing a class of problems that should naturally be modelled as instances of "common agency" (e.g. common retailers for homogeneous products produced by different suppliers, non-linear pricing of utilities, etc.), we have not yet advanced far enough. Let us return to our organizing example for a moment and point out the directions in which research still needs to progress.

First and foremost, in most situations of common retailers (and the D.J. in the Coasean payola example is just an instance of a retailer, albeit a special case), the retailer has private information about the market, or a particular customer. For instance, the travel agent knows the local market for package holidays better than the tour operator (who might want to design a product on the basis of the agent's advice); the independent financial adviser has more information about a particular client's background (e.g. health status) or his preferences (e.g. preferred payment schedule for life assurance products) than the insurer who offers the product (and who will want to design a policy such that it fits the customer); and the D.J. knows the tastes of her audience (and accordingly record producers may or may not want to advertise to this market through payola). In all these cases, in one way or another, principals interact on a product market. Importantly, however, producers want to learn a characteristic of the market, because this characteristic (e.g. the anticipated popularity of their record) will influence their payoff. In other words, we want a model in which producers' utilities depend, not only on the output (the decision) of each of her rivals, but also on the market characteristics she seeks to learn from the agent. Further, in many situations of economic interest (the payola case, the package holiday case, and the insurance contract example, are instances of this) the product sold is a differentiated product, and we could interpret $\theta$ as information about any product's location in the space of consumer preferences.

We might want to set up a model like this in the following way: let each principal (record producer) $i$ have a record (record $i$) which she seeks to introduce to the market; let $x_i$ be the frequency with which producer $i$'s record is played on a D.J.'s radio show. The D.J.'s (agent's) utility then is $u(x_1, x_2, \theta) + t_1(x_1) + t_2(x_2)$, where $\theta$ measures the relative popularity of record 1, such that, $u_{x_1\theta} > 0$ in the usual way, but of course $u_{x_2\theta} < 0$. This reflects the fact that the D.J.'s popularity is influenced by the records she plays, and, while more records are better than less (several radio stations now advertise "more music, less chat"), the marginal gain from playing more of a record increases with the record's popularity. A natural place to start the modelling would be further to assume that $u_{x_i x_{-i}} > 0$ (audience likes variety). Further, in the payola case, payments are restricted to be of the form $t_i(x_i) = \tau_i x_i$ (i.e. payment schedules are linear), although it is not clear whether this feature should be imposed on the model at the outset. Each producer's utility $v^i(x_1, x_2, \theta)$ depends on the number of times its record and its rival's record is played, as well as on the record's popularity. One should expect that $v^i_{x_i} > 0$, and $v^i_{x_i x_{-i}} < 0$, and that $v^1_{x_1\theta} > 0$ while $v^2_{x_2\theta} < 0$.

Proceeding in the usual way (holding principal 2's mechanism fixed), we could find principal 1's optimal response to principal 2's contract offer. Then holding principal 1's contract fixed, and defining $\tilde{\theta} = 1 - \theta$, we could do the same for principal 2 (the redefinition of $\theta$ serves to make the constraint reduction theorem applicable). The presence of $\theta$ in each principal's payoff function will not affect the solution technique.

What will the solution look like? Producer 1 seeks to design a schedule such that the D.J. prefers playing a lot of record 1 when $\theta$ is high. Therefore, she has to leave the agent increasing rent when $\theta$ is high. But producer 2 seeks to design a schedule such that the D.J. prefers playing a lot of record 2 when $\theta$ is low, so she has to leave the agent higher rent when $\theta$ is low. If principal 1 were to increase the number of times with which her record is played, since $u_{x_i x_{-i}} > 0$ (records are complements), the agent would want to play more of principal 2's record. This would reduce her rent (from principal 2), so that there is less rent for principal 1 to extract, and so principal 1's incentive to increase the number of times her record is played is reduced. It is not clear therefore that (as in the case of substitutes in the common agency game of Martimort (1992) and Stole (1992)) there is either over or underprovision of $x_i$, relative to second-best. There is another incentive that works in the same direction: if principal 1 increases the number of times her record is played (and therefore increases the chances that her records are sold), she will increase (because records are complements) the number of times her rival's record is played. This in turn reduces her chances for record sales (we expressed this by assuming that $v^i_{x_i x_{-i}} < 0$), so that her incentive to increase the number of times her record is played is reduced.

Note that we have assumed record popularity to be fixed: the question was one of introducing a given record to the market. Further complications arise when principals use a common agent to acquire information about a customer's preferences so that they can design a product specific to that customer. This is what we might want to model in the insurance sales context: rival insurers attempt to obtain information about the customer's preferences from an insurance broker (and want the broker to misrepresent that information to the other principal), so that they can offer the customer the "right" (for the customer) product, while the rival offers the "wrong" (for the customer) product. In equilibrium, if the revelation principle is applicable, the agent will report correctly to both principals (so that the customer obtains the "right" policy), but this will generally come at a rent cost.

### Horizontally Differentiated Principals

Mezzetti (1997) goes some way to address the issue of horizontal differentiation of principals. In a very specialized model, with fully specified agent utility and uniform distribution over the type space, Mezzetti studies two principals with horizontally differentiated products who use a common (sales-) agent. The agent has private information about her marginal productivity in selling the two products (assumed to be complements for consumers). The agent, of course, faces countervailing incentives: since equilibrium reports to both principals coincide, if she over-reports to one principal (i.e. extracts rent), she under-reports to the other principal (i.e. obtains reduced rent). In general, one should therefore expect lower levels of rent extraction in common agency than when principals co-operate (as in the benchmark case, above). Mezzetti also shows that over an intermediate type space (the agent is similarly productive in her sales performance for both products), there is pooling of agent types (that is, a flat transfer schedule). One should therefore expect commission payments for specialized sales agents and flat fees for agents who are similarly productive in both tasks. The intuition for pooling of types, of course, comes from countervailing incentives: while reporting a higher type to one principal is beneficial to the agent (in terms of rent extraction), she loses rent from the other principal. If the level of activity in both tasks is similar (because the agent is of similar productivity in each), the transfer schedule in this region is likely to be flat.

While clearly a specialized model, Mezzetti (1997) studies an important practical question. However, we still have no answer to our motivating question of the interaction of differentiated products, through a retailer, on a market on which the two products are close substitutes.

The common agency literature has not had as large an impact as many have hoped. Much of this is due to the difficulties we have pointed out above: the general inapplicability of the revelation principle, and the fact that the theoretical literature has not been generalized sufficiently to address problems of economic concern such as those we have just outlined. Yet, some applied work has been carried out; and we review some of it below. First, however, we present a closely related class of models, in which two principal-agent hierarchies compete against each other. There are obvious, close links with the literature on common agency, and we will find a brief look at the model's methodology useful in order to classify applied work.

## 4.6 Competing Hierarchies

In a class of models intimately related to common agency models, one-principal one-agent hierarchies compete against each other. Consider, for instance, the literature on tax competition: government-firm pairs in two countries compete for export sales to a third country (as in Brainard and Martimort (1996)). Alternatively, a natural question to ask is: what is the obvious alternative to common sales agency? The obvious answer is: exclusive dealing. From the perspective of "competing hierarchies" models, exclusive dealing is the competition of firm-exclusive dealer pairs in a downstream market (Martimort (1996a)). An early literature survey is Gal-Or (1997). Here, we focus instead, as indeed we have done for most of this chapter, on modelling technique.

### 4.6.1 The Model

We use the same notation as above, with only minor (obvious changes). Let there be two agents $i = 1, 2$ of type $\theta$, and two principals $i = 1, 2$. Principal $i$ offers a take-it-or-leave-it contract $(x_i(\hat{\theta}_i), t_i(\hat{\theta}_i))$ to agent $i$ (considering direct revelation mechanisms is without loss of generality when contracts are private). With only slight changes of notation, denote agent $i$'s indirect utility when she reports $\hat{\theta}_i$ by $U_i(\hat{\theta}_i, \theta) \equiv u^i(x_1(\hat{\theta}_1), x_2(\hat{\theta}_2), \theta) + t_i(\hat{\theta}_i)$, and principal $i$'s utility as $v^i(x_i(\hat{\theta}_i)) - t_i(\hat{\theta}_i)$. As before, we first characterize implementability and then proceed to use the revelation principle to derive best responses.

### 4.6.2 Solving the Model

Consider first implementability. We know that in a Nash equilibrium for agent 1, $U_1(\theta) \equiv U_1(\theta, \theta) = \max_{\hat{\theta}_1} u^1(x_1(\hat{\theta}_1), x_2(\theta), \theta) + t_1(\hat{\theta}_1)$, and similarly for agent 2. By the envelope theorem we obtain immediately

$$U_1'(\theta) = u^1_{x_2}(x_1(\theta), x_2(\theta), \theta)x_2'(\theta) + u^1_\theta(x_1(\theta), x_2(\theta), \theta) \tag{4.20}$$

which is not straightforwardly increasing in type. However, the literature has usually proceeded by placing conditions on $u$ that are sufficient for $U_i' > 0$, so that linear programming techniques may be used.[22] Equation (4.20) implies obviously that $t_1'(\theta) = -u^1_{x_1}(x_1(\theta), x_2(\theta), \theta)x_1'(\theta)$. The local second order condition is

$$\left(u^1_{x_1 x_2}(x_1(\theta), x_2(\theta), \theta)x_2'(\theta) + u^1_{x_1\theta}(x_1(\theta), x_2(\theta), \theta)\right) x_1'(\theta) \geq 0.$$

Principal 1's maximization problem is to

$$\max_{x_1(\cdot), U_1(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \left(v^1(x_1(\theta)) + u^1(x_1(\theta), x_2(\theta), \theta) - U_1(\theta)\right) f(\theta) d\theta,$$

subject to (4.20) and $U_1(\underline{\theta}) = 0$ (assuming that $U_1'(\theta) > 0$). The Hamiltonian for this

[22] By restricting the agent to "aggregated" payoff functions, i.e. payoff functions $u(\cdot)$ that satisfy $\frac{\partial}{\partial x_1}\left(\frac{u_{x_2}(x_1, x_2, \theta)}{u_\theta(x_1, x_2, \theta)}\right) = 0$, increasing rent, concave Hamiltonian, and global optimality of the principal's problem can be ensured (cf. Martimort (1996a)).

problem is

$$\begin{aligned} H(x_1,U_1,\theta,\lambda) \;\equiv\; & \left(v^1(x_1(\theta)) + u^1(x_1(\theta),x_2(\theta),\theta) - U_1(\theta)\right) f(\theta) \\ & +\lambda(\theta)\left(u^1_{x_2}(x_1(\theta),x_2(\theta),\theta)x_2'(\theta) + u^1_\theta(x_1(\theta),x_2(\theta),\theta)\right) \end{aligned}$$

This yields the following necessary condition for a differentiable equilibrium for principal $i$:[23]

$$v^i_{x_i} + u^i_{x_1} = \frac{1-F(\theta)}{f(\theta)}\left(u^i_{x_1x_2}x_j' + u^i_{x_1\theta}\right). \tag{4.21}$$

As before, the properties of the equilibrium solution depend on a term $u_{x_1x_2}$, and therefore on whether the goods are substitutes ($u_{x_1x_2} < 0$) or complements ($u_{x_1x_2} > 0$). When the contract decisions $x_1$ and $x_2$ are substitutes, there exists a unique equilibrium, with output between first-best and the output achieved if the principal-agent hierarchies did not compete (i.e. the standard principal-agent model), and when decisions are complements, a continuum of equilibria with output below that achieved in the standard principal-agent model (Martimort (1996a, Proposition 2)).

## 4.7 Applications

This section discusses some straightforward applications of the general common-agency theory. Much of the applied work in this area follows closely the theoretical model of Martimort (1992) and Stole (1992).

### 4.7.1 Common Marketing Agents

In an adverse selection context, the general theory developed in Martimort (1992) and Stole (1992) can be straightforwardly applied to a model of common marketing agents. In a worked example (Martimort (1992)), the agent faces a set of inverse demand schedules for the products of both principals and has a (uniformly distributed) marginal cost, $\theta$. The agent's utility function therefore becomes $u(x_1,x_2,\theta)+t_1+t_2 = (a-\theta)(x_1+x_2)-\frac{b}{2}(x_1^2+x_2^2)+$

[23] Conditions need to be placed on $x_1'$ to ensure differentiability of the equilibrium.

$2cx_1x_2 + t_1 + t_2$. The principals' objective is to minimize monetary transfers, i.e. $v^j(\cdot) = 0$. The form of the problem allows an explicit solution for the symmetric equilibrium decisions, but otherwise does not add to the results from the general model.

Martimort (1996a) models the choice of two principals (producers) between two organizational structures: a common marketing agency (common agency) and exclusive dealing (competing hierarchies). The timing of the game is the following: First, principals choose retailing structures. In the continuation game, decisions (and principals' utilities) are determined according to equations (4.19) and (4.21). The setting is the same as that in Martimort (1992), with inverse demand curves $p_j = a - \frac{b}{2}x_j + cx_i$ for $j = 1, 2$, $j \neq i$, and the agents' privately known marginal cost $\theta$, distributed uniformly on $[\underline{\theta}, \overline{\theta}]$. (Alternatively, $\theta$ could be interpreted as the agent's private information about the state of demand.) Under common agency, the agent maximizes $u^{ca}(x_1, x_2, \theta) + t_1 + t_2 = (a - \theta)(x_1 + x_2) - \frac{b}{2}(x_1^2 + x_2^2) + 2cx_1x_2 + t_1 + t_2$; in the competing hierarchies model, each agent $j$ maximizes $u^{ch}(x_1, x_2, \theta) + t_j = (a - \theta)x_j - \frac{b}{2}x_j^2 + cx_jx_i + t_j$. Again, the principals' objective is to minimize monetary transfers, i.e. $v^j(\cdot) = 0$. The principals' choice of marketing structure reduces then to a comparison of principals' utilities under common agency and competing hierarchies. Roughly, in the common agency model, the level of upstream coordination is high, but the agent can misrepresent her private information to two principals, which introduces the additional inefficiency (externality in rent extraction) discussed above. In the competing hierarchies model, this inefficiency is removed, but coordination between principals is low.

### 4.7.2 The Internal Organization of Government

Martimort (1996b) presents a simple and straightforward application of the common agency model under adverse selection. Let the agent be a firm with production cost $\theta$, and let the decision be a choice between production and non-production. (In terms of the general model, the decisions are contract complements.) Principal $j$ (a regulator) maximizes social benefit $S_j$ in her jurisdiction and is restricted to paying lump-sum transfers $T_j$ to the agent. Transfers are socially costly, i.e. they have a shadow cost $\lambda$. Principal $j$'s expected utility is therefore

$$(S_j - (1+\lambda)T_j)F(T_1+T_2) + \int_{\underline{\theta}}^{T_1+T_2} (T_1+T_2-\theta)f(\theta)d\theta.$$

In the cooperative (integrated) case, where the (single) principal's concern is with $S_1+S_2$, and letting $T = T_1 + T_2$, we can calculate the cut-off value of $\theta$, $\theta^I$, below which the project is performed. $\theta^I$ solves $S_1 + S_2 = (1+\lambda)\theta^I + \lambda\frac{F(\theta^I)}{f(\theta^I)}$. Under non-cooperation (separation), the cut-off value $\theta^S$ solves $S_1 + S_2 = (1+\lambda)\theta^S + 2\lambda\frac{F(\theta^S)}{f(\theta^S)}$. A generalization to $n$ principals yields a cut-off $\theta^N$ that solves $\sum_{j=1}^{n} S_j = (1+\lambda)\theta^N + n\lambda\frac{F(\theta^N)}{f(\theta^N)}$. Obviously, $\theta^S < \theta^I < \theta^F$ (where $\theta^F$ is the full-information cut-off value) and the inefficiency increases as the number of principals $n$ increases. (Note that, for the complements case, the general model predicts "too much" rent reduction, i.e. a decision that is "too low".)

### 4.7.3 Strategic Trade Policy Design

The general theory on competing hierarchies under adverse selection developed in Martimort (1996a) is used, with only slight change of notation ($u(\cdot)$ is specified as profit from selling output in a third country, with a simple linear inverse demand curve), in Brainard and Martimort (1996) to model the design of trade policy. Two government-firm pairs compete against each other for exports to a third country. Firms compete in quantities on the product market, while governments can make transfers (pay subsidies) to the firm in their country. In this model, there exists a multiplicity of (symmetric) equilibria, which furthermore can be ranked with respect to their output levels: output will be between the Cournot and Bertrand outputs.

Bond and Gresik (1996) model competition between two countries in tax/subsidy schedules (conditional on intra-firm trade) to a multinational corporation with a parent division located in the home country, and a subsidiary located in the host country. The parent produces an intermediate input while the subsidiary uses the input to produce final output. The home country is interested in maximizing tax income and a share of profits (say, because they may also be taxed, although this is not modelled). The foreign country seeks to maximize consumer surplus net of taxes and subsidies. Governments are incompletely informed about the firm's marginal production cost. Bond and Gresik study the two cases in which the firm makes the same cost report to both governments, and the case where the firm can make separate reports. When the cost report is joint, there exists a continuum of differentiable equilibria, all of which imply output quantities below the full-information and the co-operative benchmark cases. The intuition for this is that total welfare is determined by taxes, and split up between countries using lump-sum subsidies to the firm. The multiplicity of equilibria is supported by the multiplicity of possible divisions of the surplus between countries. With separate cost reports, when cost is distributed uniformly and demand is linear, the same result obtains.

### 4.7.4 Merger Policy

Without a clear theoretical framework, Smets and Van Cayseele (1995) address the issue of merger policy on cross-country mergers in a common agency framework. They ask the question of whether merger control should be carried out by (competing) national authorities, or by a supranational (for instance, European) competition authority. Although an important question, absent clear theory their suggestions are ambiguous, and uninformative. As our chapter 7 demonstrates, the answer (although in the simpler context of international price regulation) is far from obvious. In that chapter we study an issue similar to that addressed by Smets and Van Cayseele (1995), but provide a clear theoretical framework within which to address the problem.

# Chapter 5

# What Should the State Buy?

## With Paul A. Grout and Maija L. Halonen

In this chapter we study the provision of incentives in a three-tier hierarchy. In a very general formulation of the model, a principal delegates decisions to two agents. Agent 1 takes an unobservable action choice, linked to an outcome through a stochastic technology. This outcome is observable by agent 2, and determines that agent's type. The principal cannot observe the outcome of agent 1's action choice and therefore, by implication, cannot observe agent 2's type. Since the principal cares both about the action taken by agent 1 and agent 2's type (for instance, because agent 1's decision determines the cost with which agent 2 produces an input into the principal's production), the principal will generally wish to contract with both agents. We study a model in which institutional factors constrain the contracts that the principal can write, and we investigate the incentives for agents, given these institutional constraints.

A similar hierarchical structure has been studied by Tirole (1986) in the context of an agent who takes an unobservable (to the principal) decision under conditions of moral hazard, and a supervisor who obtains scrambled information about the agent's performance. The principal's problem in Tirole's model is to provide the correct incentives for the agent when agent and supervisor can collude to hide unfavorable information from the principal. In Tirole's paper, the principal has little interest in the supervisor's action apart from its informational implications (the supervisor's actions reveal information about the agent's performance). In our model, by contrast, we focus on the existence of both a situation of moral hazard in the relationship with agent 1, and an adverse selection problem between principal and agent 2. The principal has a direct interest in agent 2's decision, quite apart from that decision's information-revealing qualities (i.e. revealing of agent 1's action choice).

We analyze this hierarchical structure within the context of the problem of public service provision. The principal (the state) delegates service provision to agent 2 who uses an asset provided by agent 1. In a model of asymmetric information about the endogenously determined quality of the asset, we study the quality of the public service provided by agent 2 who uses the asset as an input into production of the service output. In particular, we are interested in the incentives for the producer of the asset to invest in asset quality, and the resulting quality of service provision under different, institutionally determined, ownership structures. Incentives for agent 1 will generally depend on whether the principal procures (and owns) the asset, or whether procurement (and ownership) is delegated to agent 2. We also study the case where agent 1 and agent 2 are integrated, so that one agent both provides the service output and takes the investment decision. We find that in this latter case ownership of the asset by the agent (rather than by the principal) tends to lead to higher-powered incentives to invest in asset quality precisely when the public service is "essential," that is, when its demand is inelastic.

The reason for our interest in this model is the growing importance of private sector involvement in public service provision and capital investment. Despite the importance of public sector involvement in public procurement, we are not aware of an argument that favors private over public investment. In this paper, we provide such an argument. Our recommendation is that, under a wide range of circumstances, the state should buy services only, and allow service providers to procure assets. This justifies the present UK government's dedication to the private financing of capital assets for public service provision.

## 5.1 Introduction

Since the work of Buchanan and Tullock (1962), we know that the traditional view of governments as benevolent public good providers, best placed to deliver efficient public services, is mistaken. In practice, in many countries the boundaries of the state have been redefined: the role of governments has shifted from provider of public services to designer of market mechanisms for the private provision of these services. Examples of this redefinition of the state are the competitive tendering for public service provision, the privatization and regulation of public utilities, education voucher schemes, the design of auctions for the allocation of the radio spectrum or broadcasting licences, and so on. The Private Finance Initiative (PFI) in the UK is just one example of this ubiquitous trend.

The aim of the Private Finance Initiative is to transform the public sector

> "from being an owner of capital assets and direct provider of services, into a purchaser of services from a private sector partner responsible for owning and operating the capital asset that is delivering the service. ... [C]onventionally, a builder puts up a structure, takes his fee and moves on. It is not the builder or designer who has to live with the asset. The public sector is left to manage an asset that they are unlikely to be best placed to operate. A PFI deal harnesses the operating expertise of the private sector contractors involved in the design and build stages ..." HM Treasury *Economic Briefing* 9 (April 1996)

Clearly, this is an important trend. In the UK, during the fiscal year 1997–1998, investment under the private finance initiative amounted to £2 billion of a total procurement budget of £13 billion. Given the present government's dedication to PFI and public-private partnerships, a sustained increase in the proportion of privately financed capital investment seems likely. It is therefore important that we understand the different incentives for builders of assets and public service providers that are created by PFI. We do not find the reason of "greater operating expertise of the private sector" *per se* compelling.

We have just cited non-benevolence of government as one reason why public provision of public services may not be optimal. But non-benevolence is just one reason to doubt whether public service provision is best undertaken by the state. Even benevolent governments may not be in the best position to guarantee quality of public service provision, for instance, when quality is only imperfectly observable, or when complete contracts specifying the service output cannot be written. In the case of contractual incompleteness, different ownership patterns induce different incentives for provision of quality in the public service.[1,2] For instance, Hart, Schleifer, and Vishny (1997) study the question of public versus private ownership when investments in cost reduction and investments in quality improvements cannot be contracted on (say, because contracts cannot precisely define the actions to be taken in every conceivable circumstance). Generally, they find that incentives for cost reduction and quality improvement are weak under public ownership and strong under private ownership of the assets used to provide the public service. Whether public or private ownership is to be chosen then depends on the trade-off between cost reduction and the implications this has for service quality.

In this chapter, we also study the incentives for quality in asset and service provision under different ownership structures. In particular, we are interested in the incentives for quality in asset and service provision under PFI, and we contrast them with the incentive structures under traditional public-sector management of state-owned assets. However, we do not rely on incomplete contracting as the driving force behind our results. In fact, we show that results similar to those of Hart, Schleifer, and Vishny (1997) can be obtained even when complete contracts can be written.

In our model, an asset necessary for the provision of a public service is to be procured (examples of this are prisons, hospitals, schools, etc.). Once the asset is built, government cannot observe the quality of the asset which is then used to provide a public service, in the following sense: the state cannot observe whether a private builder has made a cost-reducing investment in the asset's building phase. Investment in asset quality reduces the cost of service provision and therefore increases the quantity, or quality, of the service output. Governments may well be in a position to write complete contracts with the service provider, that is, a contract fully revealing of cost conditions. Once the government knows the cost of service provision, it can then "back out" the likelihood with which the builder has invested in cost reduction (i.e. asset quality), and can then impose sanctions on the builder. Typically, these sanctions are damages imposed by a court, and previously specified in the contract the government writes with the builder. In equilibrium, these sanctions should be chosen such that the builder makes the cost-reducing investment. However, this result does

[1] The literature on incomplete contracts begins with the paper by Grossman and Hart (1986), and Hart and Moore (1990), and is summarized in Hart (1995).

[2] For a survey of the literature on public versus private ownership, cf. Shleifer (1998).

not necessarily hold.

The intuition for our results is best expressed as a commitment problem. Typically, the state would wish to write a (complete) contract with the service provider that induces the service provider to provide high service quality (or high quantity) of output when the cost of service provision is low (and the per-unit price the state pays for the service output is therefore correspondingly low) and *vice versa*. From the information obtained in this "revealing" contract, the state can then provide incentives to the builder (via damage payments) to invest in cost reduction (or high asset quality). However, when governments have a concern both for the consumer surplus created through provision of the public service and an incentive to maximize monetary income (for instance, because this can be used to finance other public projects), the fact that damages can be imposed on the builder may act as an incentive for the state not to want to find the cost of service provision. Suppose that the chance of obtaining damage payments in court is greater the higher the price paid by the state to the private provider of the public service. If the damage payments the state can win in court are high enough, the state may wish to pay a high price for the service output always, claim that no cost-reduction investment was carried out by the builder, and obtain damage payments in court.

Clearly, there is an externality that arises when the state can obtain damage payments for (alleged) non-performance. Since the state cannot commit not to "free ride" on the builder (by extracting damage payments from her), investment will generally be low. If the state could commit not to change its service contract with the service provider (i.e. to misrepresent cost as being high), the problem would disappear. In general, though, there is no mechanism for this commitment when the state owns (and procures) the asset.

However, the government can transmit ownership (and procurement) of the asset to the service provider. We study two cases, according to whether the service provider builds the asset herself or contracts out the building of the asset. In the latter case of an "arm's length" relationship between the state and the builder via the service provider, similar incentives to extract damages exist, although this time the service provider benefits directly from the damages, and this indirectly reduces the cost to the government of contracting with the service provider. In general, we should expect no difference between this case and that of public ownership when complete contracts can be written. The more interesting case, however, is that where the service provider also builds the asset. In this case, the externality (extraction of damages from the builder) is internalized: the service provider (who now *is* the builder) cannot sue itself for damages. We show that when the service provider owns and builds the asset, typically higher levels of cost-reducing investment can be implemented than under public ownership. We also show that this conclusion hinges on the nature of demand for the public service. Finally, we study the question of choice between different ownership structures when private ownership induces greater investment. In this case, we find that for projects for which cost can be reduced relatively cheaply, state ownership of the asset remains optimal. However, when the cost of investment in asset quality (cost reduction) is either very low or very high, private ownership of the asset becomes optimal for the state.

This justifies our recommendation that, for a large class of circumstances, what the state should buy are services, not assets.

The chapter is organized as follows. Section 5.2 outlines the model. Section 5.3 studies the different contracts that could be used by the state to contract on the delivery of the public service by the private service provider. In section 5.4 we use these contracts to compare the implications for investment under different ownership structures. Different ownership structures will make different contracts optimal, and this allows us to make judgements about the implications for investment in cost reduction under different ownership structures. We also address the state's problem of choice between different ownership structures. Finally, section 5.5 concludes.

## 5.2 The Model

A principal (the state) seeks to procure an asset necessary to provide a public service. The principal delegates service provision to a service provider. The asset is produced by a builder who owns private information about asset quality. The builder obtains a fixed fee for building the asset. We assume that bidding for building contracts is by competitive tender, and that there is a large number of potential builders, so that the builder's fixed fee will just cover the builder's expected costs.

The asset may be of high or low quality. Without loss of generality, let asset quality be the unit cost of service provision for the service provider, $\theta \in \{\underline{\theta}, \overline{\theta}\}$, with $\overline{\theta} > \underline{\theta}$. Denote high asset quality as low unit cost of service provision ($\underline{\theta}$) and low quality as high unit cost of service provision ($\overline{\theta}$). Define $\Delta\theta = \overline{\theta} - \underline{\theta}$. The builder, at a cost (investment in asset quality) $i$, controls the distribution over asset qualities, and the size of $i$ is common knowledge. If the builder does not make the investment $i$, the distribution over asset qualities is $\{(\underline{\theta}, p_0), (\overline{\theta}, (1-p_0))\}$. If she does make the investment, the distribution is $\{(\underline{\theta}, p_1), (\overline{\theta}, (1-p_1))\}$, with $p_1 > p_0$. We make the following modelling assumption. There are three states of nature: in state 1, $\theta = \underline{\theta}$ for certain; in state 2, by default, $\theta = \overline{\theta}$, but the builder has the option to pay $i$ and raise quality to $\underline{\theta}$; and in state 3, $\theta = \overline{\theta}$ for certain. After the builder has carried out any investments, the service provider privately learns the resulting cost of service provision $\theta$ before signing its (service provision) contract with the principal.

The demand curve for the service output is $q(\cdot)$, such that $q'(\cdot) < 0$, with inverse demand $q^{-1}(\cdot)$. We denote $\underline{q} = q(\underline{\theta})$ and $\overline{q} = q(\overline{\theta})$. Note that $\underline{q} > \overline{q}$. Define $\Delta q = \underline{q} - \overline{q}$. Importantly, this notation is neutral as to whether we choose to model the quantity or quality of service provision, and we will often find it convenient to interpret $q$ as quality of service. The service provider produces output $q$ at a total cost of $\theta q$, for which she is paid $cq(c)$ (where $c$ is the principal's conjecture about cost) and possibly obtains a subsidy $s$ from the principal. We assume that, since the support of the distribution over costs is known, the only admissible conjectures for $c$ are $c \in \{\underline{\theta}, \overline{\theta}\}$.

In some versions of our model, agents will be able to claim damages against the builder (for underinvestment in asset quality). The amount of damages that can be claimed is determined in a contract with the builder, and we denote the size of damages by $d$. Damage payments are awarded by courts. We assume that courts can observe the payment to the service provider for the service output, but not the size of the government subsidy ($s$) to the service provider. This assumption is natural: in the present setting, $s$ may be non-monetary; for instance, we may interpret $s$ as contractible effort the principal can spend on behalf of the service provider. Although such effort may be contractible for the two parties, it is not observable by the courts.

All agents are risk neutral. The principal's objective is the maximization of net consumer surplus, $v(q) - cq - s + d - m$, when she can claim damages $d$, and for a given monetary payment (for instance, to the builder) of $m$. We assume that $v''(\cdot) < 0$.[3] We also interpret $cq$, $s$ and $m$ as payments that have to be raised through taxation (and therefore they enter the principal's objective), and $d$ is revenue that can be used for other budgetary purposes.[4]

The service provider maximizes her monetary payoff $s + d$ (if she can claim damages), and has an outside utility level which we normalize to zero. When the builder obtains a fixed fee $m$, the builder's objective is $m - d$ (if damages are claimed), and her outside utility level similarly is normalized to zero.

Our aim is to compare incentives for quality in asset building and service provision under two ownership structures:

### 5.2.1 "Traditional" public service provision

The principal owns the asset. The principal hires a builder to build the asset. Building contracts are awarded competitively. This contract specifies a damage payment $d$ that may be imposed if the principal seeks recourse to the courts, and a flat fee to the builder, $m$. Asset quality is not observable by the principal. A service provider, different from the builder, provides service output $q$ at unit cost $\theta$. The quantity (or quality) of service output $(q)$, and the service provider's reward, $s$, is determined through a contract between principal and service provider. In particular, the principal has a choice between writing an incentive compatible (fully revealing of cost conditions) contract, or a non-revealing contract. In either case, the contract has to allow the service provider a nonnegative rent, whatever the cost conditions may be. This assumption captures the fact that the public service is necessary: the service provider can not be allowed to quit. Whether the principal has chosen a revealing or a non-revealing contract, she can attempt to sue the builder for not investing in quality improvement. Damages of uniform size $d$ (i.e. damages that do not depend on the size of the actual or claimed loss) will be awarded by courts when $c = \overline{\theta}$, and no damages will be awarded when $c = \underline{\theta}$. That is, when the principal pays the lowest possible price for the service this is reason for courts to believe that asset quality must have been high, so

[3] The assumption that $v''(\cdot) < 0$ flows naturally from our interpretation of $v(q)$ as gross consumer surplus. Consumer surplus, for an inverse demand function $q^{-1}(q)$, is $v(q) = \int_0^q q^{-1}(t)dt$. Obviously, $v''(q) = \frac{d}{dq}q^{-1}(q) < 0$.

[4] We also assume that the principal has no concern for the builder's (or service provider's) welfare. Clearly, this is extreme. However, as long as the weight on the builder's welfare in the principal's utility function is less than unity, our qualitative results will still obtain.

that damages are not awarded; conversely, when courts observe a high price for the service output, damages will be awarded.[5,6] The potential gain to the principal from writing a non-revealing contract is that although it generally involves higher payments to the service provider, these payments allow the principal to win damages with greater probability. We assume that courts can only observe the price paid to the service provider and the quantity (or quality) of the service provided. For reasons pointed out above, courts cannot observe the subsidy to the service provider. This implies that when courts observe a price $\overline{\theta}$ or corresponding quantity $\overline{q}$, this is uninformative about whether the principal has written a revealing or non-revealing contract with the service provider.

### 5.2.2 PFI

The service provider owns the asset, and either builds the asset herself, or procures the asset privately. The principal contracts with the service provider on the quantity (quality) and the price of the service output, and specifies any subsidies to the service provider. Since the principal no longer owns the asset, she has no recourse to the courts if she suspects underinvestment in asset quality. In the private procurement case, where the service provider contracts out the provision of the asset to a builder, the service provider can write a damage contract (enforceable by the courts) with the builder of the asset. In the case in which the service provider herself builds the asset, the principal allows the service provider to build the asset that she then uses to provide the service.

First, we study the characteristics of revealing and non-revealing contracts. We then turn to an examination of incentives for providing high asset quality, and how incentives for asset quality, and incentives for misrepresentation of actual cost are determined by the size of damage payments.

[5] Here it does not matter whether $d$ is the actual or expected damage payment. Courts may well impose damages with some (fixed) probability. Our framework allows for this interpretation.

[6] In fact this strategy (award damages when price is high, do not award damages when price is low) is an equilibrium strategy for the court in the game between principal, builder, and the court, when the court can choose any strategy conditional on the observed service output. An interesting question is what happens if the court's strategy space is enlarged. For instance, from the size of the required investment $i$, the court can draw conclusions similar to those we will draw about whether investment can (and will) be induced. It seems likely that this changes the nature (and existence) of equilibria in the contract-setting game. This question, while beyond the purpose of the current paper, is sufficiently interesting to warrant future research.

## 5.3 Contracts for Service Provision

In this section, we derive the properties of the service provision contracts the principal can write with the service provider. We use these results in the following section to study the implications of asset ownership on investment in asset quality.

### 5.3.1 The Optimal (Second-Best) Revealing Contract

The second-best contract between principal and service provider is standard (cf. Baron and Myerson (1982)).[7] The revelation principle (e.g. Myerson (1982)) allows the principal to restrict attention, without loss of generality, to direct revelation mechanisms (in which the agent reports, truthfully, on her type). When the distribution over $\theta$ is (known to be) $\{(\underline{\theta}, p), (\overline{\theta}, (1-p))\}$, the principal designs a contract (schedule of subsidies) $\{\underline{s}, \overline{s}\}$ for the service provider so as to

$$\max_{\underline{s}, \overline{s}} p\left[v(\underline{q}) - \underline{q}\underline{\theta} - \underline{s}\right] + (1-p)\left[v(\overline{q}) - \overline{q}\overline{\theta} - \overline{s} + d\right] \tag{5.1}$$

s.t.

$$\underline{s} \geq \overline{q}\overline{\theta} - \overline{q}\underline{\theta} + \overline{s} \tag{5.2}$$

$$\overline{s} \geq \underline{q}\underline{\theta} - \underline{q}\overline{\theta} + \underline{s} \tag{5.3}$$

$$\underline{s} \geq 0 \tag{5.4}$$

$$\overline{s} \geq 0 \tag{5.5}$$

Constraints (5.2) and (5.3) are the incentive compatibility constraints for low and high cost providers, and (5.4) and (5.5) are the individual rationality, or participation, constraints.

[7] Note, however, that here we reduce the number of the principal's instruments: she can only control the subsidy to the service provider; the government's conjecture over cost is $c(\theta) = \theta$, and correspondingly output is $q(\theta)$.

Note that $\underline{q}$ and $\overline{q}$ are determined by $\theta$ and are not choice variables for the principal. Note also that the solution to the program (5.1)–(5.5) yields the principal's gross payoff, before any direct transfers $m$ are made.

As is standard, we have the following lemma:

**Lemma 41** *(5.2) and (5.5) are binding in equilibrium, and (5.3) and (5.4) are slack.*

**Proof.** The proof for this "constraint reduction theorem" is standard and therefore omitted here.[8] ■

Second-best subsidies are then characterized by $\overline{s} = 0$ and $\underline{s} = \overline{q}\Delta\theta$.

The principal's value function given these subsidies is:

$$V_R(p,d) = p\left[v(\underline{q}) - \underline{q}\underline{\theta} - \overline{q}\Delta\theta\right] + (1-p)\left[v(\overline{q}) - \overline{q}\overline{\theta} + d\right]$$

### 5.3.2 A Non-Revealing Contract

Alternatively, the principal may write a non-revealing contract. The incentive compatibility constraints are therefore redundant; only the individual rationality constraints are important. There are two types of contracts the principal can write. Either she can pay $c = \overline{\theta}$ per unit of output and contractually require the agent to produce output $\overline{q}$. In this case she needs to pay no subsidies to fulfil the individual rationality constraints. We refer to this contract as the high-price non-revealing contract. Alternatively, she can pay $c = \underline{\theta}$ and require output $\underline{q}$ (and in this case, needs to pay a subsidy in order to keep a potential high-cost agent above the reservation utility level). We refer to this contract as the low-price non-revealing contract.

In the low-price non-revealing contract, the per-unit price of output is $c = \underline{\theta}$, the subsidy is $s = \underline{q}\Delta\theta$, and of course no damage payments can be claimed. The principal's value function is $V_L = v(\underline{q}) - \underline{q}\underline{\theta} - \underline{q}\Delta\theta$.

In the high-price non-revealing contract, the per-unit price of output is $c = \overline{\theta}$, the subsidy is $s = 0$, and the principal has recourse to the courts, so that her value function is $V_H(d) = v(\overline{q}) - \overline{q}\overline{\theta} + d$.

[8] For a generic proof, cf. chapter 4.

**Lemma 42** *For the principal, the high-price non-revealing contract dominates the low-price non-revealing contract.*

**Proof.** We need to prove that $v(\overline{q}) - \overline{q}\overline{\theta} + d \geq v(\underline{q}) - \underline{q}\underline{\theta} - \underline{q}\Delta\theta$. (We prove this by showing that $v(\overline{q}) - \overline{q}\overline{\theta} \geq v(\underline{q}) - \underline{q}\underline{\theta} - \underline{q}\Delta\theta$. Since $d \geq 0$, this proves the lemma.) This implies, and is implied by, $v(\underline{q}) - v(\overline{q}) \leq \Delta q\overline{\theta}$. Dividing both sides by $\Delta q$ and taking limits as $\Delta q \to 0$, we have $v'(q) \leq \overline{\theta}$. Recall that $v'(q) = q^{-1}(q)$, which is less than (or equal to) $\overline{\theta}$ for all feasible values of $q$. ■

We can therefore neglect the low-price non-revealing contract in what follows. This also aids expositional clarity: we will henceforth refer to the high-price non-revealing contract just as the non-revealing contract.

If damage claims are impossible ($d = 0$), we can further order the non-revealing and the revealing contracts.

**Lemma 43** *If $d = 0$, the principal prefers the revealing contract to the non-revealing contract.*

**Proof.** The proof is similar to that just given, and is therefore left to the reader. ■

As the expected damage payment $d$ increases from zero, since both value functions $V_R(p, d)$ and $V_H(d)$ are linear in $d$ (with different slopes), there exists some $d^*$ such that for all $d < d^*$, $V_R > V_H$, and for all $d > d^*$, $V_R < V_H$. In fact, $d^* = v(\underline{q}) - v(\overline{q}) - \Delta q\underline{\theta}$.

## 5.4 Asset Ownership and Investment

This section studies the incentives for investment in asset quality. We analyze the case of traditional public service provision (the principal owns the asset) first, and derive an upper limit on possible investments. We then turn to the case where the service provider owns the asset and either contracts out the building of the asset or builds the asset itself. We find simple conditions under which asset ownership by the service provider results in greater investment in asset quality than ownership by the principal.

The intuition for our results is this. Under traditional public service provision (the principal owns the asset), the principal has a choice of writing either a revealing, or a non-revealing contract with the service provider. If she writes the revealing contract, she benefits (in expectation) from higher consumer surplus since in some states of the world the quantity (or quality) of the service output will be high, but she has to leave costly rent to the service provider. Furthermore, writing a revealing contract provides an incentive for the builder to invest in cost reduction (or asset quality), because damages will be imposed when costs are high (that is, it is likely that the builder has not made the investment), and no damages are imposed when costs are low (that is, it is likely that the builder has made the investment). If the principal writes the non-revealing contract, she loses expected consumer surplus because the output quantity (or quality) will always be low; but she gains surplus because no extra rent (subsidy) has to be left to the service provider. Furthermore, after the contract with the builder has been written (and the builder has been paid), the government gains from obtaining damages. Obviously, the higher damage payments are, the more likely the principal is to want to write a non-revealing contract. But it is precisely those damage payments that provide the incentive for the builder to invest in cost reduction (asset quality) if a revealing contract is written. Therefore, some investments cannot be induced under asset ownership by the principal, because of the incentive for the principal to deviate to writing a non-revealing contract. As we show below, the same mechanism is at work when the relationship between the principal and the builder is an arm's length relationship: When the service provider owns the asset, and contracts out the building of the asset to an independent builder, the same analysis applies.

By contrast, when the service provider owns and builds the asset, the externality the principal imposes on the builder (through its damage claims) is internalized: the service provider cannot sue herself over underinvestment in asset quality. We make this notion precise below. First, we study the case of asset ownership by the principal.

### 5.4.1 "Traditional" Public Service Provision

Traditionally, a builder (who obtains a fixed payment $m$ to cover all expected costs, whether or not actually incurred) builds an asset that the service provider then uses to produce services. The timing of play is the following: First, principal and builder sign a contract $(m, d)$ specifying the builder's fee $m$ and damage payment $d$, and the builder is paid $m$. Then, the builder decides whether or not to make the quality improving investment $i$. Whether or not this investment is made is private information to the builder, but the service provider can observe the resulting cost of service provision. Finally, the principal decides whether to write the revealing or non-revealing contract with the service provider; the service provider produces service output as specified in the contract and payoffs are realized.

We analyze the game between principal and builder. Recall that the nature of the service provision contract is observable only to principal and service provider. We have argued this above: although the service output, and the price paid for it, may be observable, the subsidy from principal to service provider is not. For essentially the same reason, the builder cannot observe the contract the principal writes with the service provider, and can therefore not condition her strategy on the contract type. This implies that we can model the game between principal and builder as a simultaneous move game. Principal and builder simultaneously choose their strategies: the principal either writes a revealing or a non-revealing contract with the service provider, and the builder either invests or does not invest if state 2 realizes. The service provider functions as an information-revelation device for the principal and has no strategic role. We focus on pure strategy equilibria.

Note that if the principal writes a non-revealing contract with the service provider, there is no incentive for the builder to invest in asset quality (i.e. invest in a favorable distribution over service production costs): whether or not she invests has no impact on the likelihood of damage claims. The price of service provision is contractually fixed at $\overline{\theta}$, and therefore the expected damage payment is $d$, whether or not the builder has invested in quality.

When the principal writes a revealing contract, damage claims do provide an incentive for the builder to invest in quality: when the builder invests, the low cost (and therefore the low per-unit price, which implies no damage claims) prevails with increased probability. In state 2 (where the builder has a choice over whether or not to invest), she will invest if, and only if, $i \leq d$. (If she does not invest, her payoff will be $m - d$; if she does invest, her payoff will be $m - i$. Comparison of these two yields the expression above.)

What about the principal's incentive to write a revealing contract? Recall that, for the principal, the revealing contract dominates the non-revealing contract if, and only if, $d < d^* = v(\underline{q}) - v(\overline{q}) - \Delta q \underline{\theta}$.

In this game there are therefore three possible pure-strategy (Nash) equilibria:

1. The principal writes the revealing contract; the builder invests. This is an equilibrium

if $d \le d^*$ and $i \le d$. Net payoffs are $V_R(p_1, d)$ for the principal and $-(1-p_1)d-(p_1-p_0)i$ for the builder.

2. The principal writes the revealing contract; the builder does not invest. This is an equilibrium if $d \le d^*$ and $i \ge d$. Net payoffs are $V_R(p_0, d)$ for the principal and $-(1-p_0)d$ for the builder.

3. The principal writes the non-revealing contract; the builder does not invest. This is an equilibrium if $d \ge d^*$and $i \ge d$. Net payoffs are $V_H(d)$ for the principal and $-d$ for the builder.

Note first that if $d \ge i$ and $d \ge d^*$, no pure strategy equilibrium exists. Note further that the choice of $d$ in the contract between principal and builder determines the equilibrium in the above game. We now study the choice of contract $(m, d)$ between principal and builder.

Since there are potentially many builders bidding for the contract, $m$ is bid down so that the builder's expected payoff is just equal to her outside utility level. Since $m$ is a straightforward monetary transfer, the principal chooses the damage payment $d$ so as to maximize the joint surplus between principal and builder. Note that in terms of the joint surplus, the (not reveal, not invest) equilibrium is dominated by the (reveal, not invest) equilibrium. The principal therefore chooses a damage payment $d$ such that $i \le d \le d^*$ (i.e. she selects the (reveal, invest) equilibrium), if the joint surplus from this equilibrium is greater than the joint surplus from the (reveal, not invest) equilibrium. That is, the principal wishes to induce investment (set a damage payment $d \in [i, d^*]$) if, and only if,

$$\begin{aligned} V_R(p_1, d) - (1-p_1)d - (p_1 - p_0)i &= \\ V_R(p_1, 0) - (p_1 - p_0)i &> \\ &> V_R(p_0, d) - (1-p_0)d \\ &= V_R(p_0, 0), \end{aligned}$$

that is, if $i \le v(\underline{q}) - v(\overline{q}) - \Delta q \underline{\theta}$. This gives us our first result:

**Proposition 44** *When the principal owns the asset, investment can be induced up to an investment cost of* $i_{TRAD} = v(\underline{q}) - v(\overline{q}) - \Delta q \underline{\theta}$.

**Proof.** The proposition follows from the argument in the text, and the observation that the (reveal, invest) equilibrium exists precisely when $i \leq d \leq d^*$. ■

### 5.4.2 PFI

We now turn to the case where the service provider owns the asset, and either builds the asset herself, or procures the asset privately. Again, we focus on the incentives for quality-improving investment by the builder.

#### Asset Provision by the Service Provider

Here, we model the private finance initiative as the service provider building the asset. For the purpose of this section, we will therefore refer to the service provider as the service provider-builder. The principal contracts with the service provider-builder on quality of service provision. In particular, neither principal nor service provider-builder have access to the courts. The principal cannot claim damages for low asset quality because she does not own the asset, and the service provider cannot claim damages against herself. The principal's choice of instrument to induce investment is therefore solely a choice between non-revealing and revealing contracts, and the rents obtained by the service provider under each contract.

If the principal writes a revealing contract (about service provision) with the service provider-builder, the service provider-builder obtains rent when costs are low, and no rent when costs are high. If the principal writes a non-revealing contract about service provision with the service provider-builder, since the principal has no recourse to the courts, there is no incentive for investment in high asset quality. So we know that under PFI, where asset-building is undertaken by the service provider, only revealing contracts will be used. This is in fact a simple corollary to lemma 43.

Since in this setting subsidies (or, more precisely, the difference between subsidies for the low-cost provider type and the high-cost provider type) govern the incentive to invest in quality, the principal may find it optimal to increase the rent left to the service provider-builder, if the loss from increased rent is outweighed by the gain in an increased probability of obtaining the low realization of cost of service provision. Since (5.2) and (5.5) are binding in equilibrium, and (5.3) and (5.4) are slack, increasing $\underline{s}$ does not distort incentive compatibility, as long as $\underline{s} \leq \overline{s} + \underline{q}\Delta\theta$.

The incentive for the service provider-builder to invest (in quality improvement) is governed by the rents obtained in the service provision contract. The service provider-builder will invest if, and only if, $i \leq \underline{s} - \overline{s}$. Since increasing $\overline{s}$ is costly for the principal and does not increase the investment incentive, we know that, in any service contract, $\overline{s} = 0$. The highest rent the principal can therefore give to the service provider-builder, and still induce separation of types is $\underline{s} = \underline{q}\Delta\theta$. Note that under this revealing service provision contract, the service provider's individual rationality (participation) constraint is of course always satisfied, and no additional transfers are required.

How far is the principal prepared to increase rent if that increase induces investment? The principal's value function from increased rent $\underline{s}^*$ (if it induces investment) is $\tilde{V}_R(p_1, 0, \underline{s}^*) \equiv p_1 \left[v(\underline{q}) - \underline{q}\underline{\theta} - \underline{s}^*\right] + (1-p_1)\left[v(\overline{q}) - \overline{q}\overline{\theta}\right]$. The value function from writing the lowest-rent revealing contract (if that does not induce investment) is $V_R(p_0, 0) = p_0 \left[v(\underline{q}) - \underline{q}\underline{\theta} - \overline{q}\Delta\theta\right] + (1 - p_0)\left[v(\overline{q}) - \overline{q}\overline{\theta}\right]$. If she can induce investment that way, the principal would therefore wish to increase the subsidy to the low-cost agent up to $\underline{s}^* = \frac{p_1 - p_0}{p_1}\left[v(\underline{q}) - v(\overline{q})\right] - \frac{p_1 - p_0}{p_1}\left[\underline{q}\underline{\theta} - \overline{q}\overline{\theta}\right] + \frac{p_0}{p_1}\overline{q}\Delta\theta$. The same method of proof employed above shows that $\overline{q}\Delta\theta \leq \underline{s}^* \leq \underline{q}\Delta\theta$, so that we know that the point to which the principal would wish to increase the subsidy to the service provider-builder is (a) greater than the lowest rent that induces revelation and (b) less than the highest rent that still induces separation. This last result is summarized in the following lemma:

**Lemma 45** $\overline{q}\Delta\theta \leq \underline{s}^* \leq \underline{q}\Delta\theta$.

**Proof.** It is straightforward that $\overline{q}\Delta\theta \leq \underline{s}^*$. We need to show that

$$v(\underline{q}) - v(\overline{q}) \geq \Delta q \underline{\theta}.$$

Using the by now familiar method of proof: dividing by $\Delta q$, and letting $\Delta q \to 0$, we have

$$v'(q) \geq \underline{\theta},$$

which is true for all $q \in [\underline{\theta}, \overline{\theta}]$.

The proof that $\underline{s}^* \leq \underline{q}\Delta\theta$ is slightly more involved. We need to show that

$$\frac{p_1 - p_0}{p_1}\left[v(\underline{q}) - v(\overline{q})\right] - \frac{p_1 - p_0}{p_1}\left[\underline{q\theta} - \overline{q\theta}\right] + \frac{p_0}{p_1}\overline{q}\Delta\theta \leq \underline{q}\Delta\theta$$

or

$$(p_1 - p_0)\left[v(\underline{q}) - v(\overline{q})\right] + p_0\Delta q\underline{\theta} - p_1\Delta q\underline{\theta} - p_1\Delta q\Delta\theta \leq 0$$

again, dividing by $\Delta q$ and letting $\Delta q \to 0$, we obtain

$$v'(q) \leq \underline{\theta} + \frac{p_1}{p_1 - p_0}\Delta\theta$$

which, since $0 < \frac{p_1}{p_1 - p_0} < 1$, is true for all $q \in [\underline{\theta}, \overline{\theta}]$. ∎

The preceding discussion prompts our next result:

**Proposition 46** *When the service provider owns the asset and builds the asset herself, investment can be induced up to an investment cost of* $i_{PFI1} = \frac{p_1 - p_0}{p_1}\left[v(\underline{q}) - v(\overline{q})\right] - \frac{p_1 - p_0}{p_1}\left[\underline{q\theta} - \overline{q\theta}\right] + \frac{p_0}{p_1}\overline{q}\Delta\theta$.

**Proof.** Since we know that the service provider-builder will invest if, and only if, $i \leq \underline{s}$, and we know that the principal is willing to increase $\underline{s}$ up to $\frac{p_1 - p_0}{p_1}\left[v(\underline{q}) - v(\overline{q})\right] - \frac{p_1 - p_0}{p_1}\left[\underline{q\theta} - \overline{q\theta}\right] + \frac{p_0}{p_1}\overline{q}\Delta\theta$, the proposition follows. ∎

### Private Procurement of the Asset

Instead of building the asset herself, the service provider may contract out the building of the asset to a builder. As before, the principal has no recourse to the courts (since she does not own the asset), and her instrument to induce investment is solely the choice of service provision contract, and the amount of rent left to the builder in this contract. Now, however, the service provider writes a damage contract $(m, d)$ with the builder, and has recourse to the courts if the observed cost of service production is $\overline{\theta}$. The service provider's ability to obtain damages when observed cost is high alters the principal's mechanism design problem. The principal now seeks to:

$$\max_{\underline{s},\overline{s}} p\left[v(\underline{q})-\underline{q}\underline{\theta}-\underline{s}\right]+(1-p)\left[v(\overline{q})-\overline{q}\overline{\theta}-\overline{s}\right] \tag{5.6}$$

s.t.

$$\underline{s} \geq \overline{q}\overline{\theta}-\overline{q}\underline{\theta}+\overline{s}+d \tag{5.7}$$

$$\overline{s}+d \geq \underline{q}\underline{\theta}-\underline{q}\overline{\theta}+\underline{s} \tag{5.8}$$

$$\underline{s} \geq 0 \tag{5.9}$$

$$\overline{s}+d \geq 0 \tag{5.10}$$

Note again that this program is written before any flat fee payments $m$ are made.

We solve this program by simple change of variable. Define $\tilde{s} \equiv \overline{s}+d$. The problem then reduces to that of program (5.1)–(5.5), with second-best subsidies that are characterized by $\tilde{s} \equiv \overline{s}+d=0$ and $\underline{s}=\overline{q}\Delta\theta$. Note that the principal's objective in this new program is to

$$\max_{\underline{s},\tilde{s}} p\left[v(\underline{q})-\underline{q}\underline{\theta}-\underline{s}\right]+(1-p)\left[v(\overline{q})-\overline{q}\overline{\theta}-\tilde{s}+d\right].$$

The intuition for this result is simple: although the fact that the service provider can obtain damage payments from the builder makes incentive compatibility constraint (5.7) harder to satisfy, the high-cost provider type's individual rationality (or, participation) constraint (5.10) is now easier to satisfy. In essence, the principal can extract from the service provider the damage payment that the builder pays to the service provider.

Of course, the builder's participation decision has to be ensured, by transferring the expected cost of asset building to her. The principal therefore needs to give both service provider types enough rent to be able to pay the builder. Note that the principal's choice of this fixed transfer determines the damage payments chosen by the service provider.[9] Given the revealing contract (5.6)–(5.10), if it induces investment, the builder requires a fixed fee payment of $(p_1 - p_0)i + (1 - p_1)d$. If this contract induces investment, the principal's payoff will therefore be $V_R(p_1, d) - (p_1 - p_0)i - (1 - p_1)d = V_R(p_1, 0) - (p_1 - p_0)i$. The principal will wish to induce investment if this payoff is greater than the maximum payoff the principal can obtain through a contract that does not induce investment. It is this contract that is the relevant comparison for the principal's choice of investment.

We now address the question of which contract is the relevant contract for comparison. We know that the principal always has to ensure, albeit indirectly through the service provider, participation of the builder. That is, the principal has to transfer to the service provider the fixed fee the builder requires to satisfy her participation decision. The service provider then pays the builder that fixed fee. This implies, in particular, that although the principal can recover (through appropriate service contract design) from the service provider any damage payments the builder makes to the service provider, the principal needs to transfer the same expected payment to the builder (via the service provider) to satisfy the builder's participation constraint. In any contract, therefore, any damage payments the principal could recover from the service provider are cancelled out by transfers to the builder (via the service provider) of the same magnitude. But we already know from lemma 43 that, when the damage payments the principal receives are zero, the principal prefers a revealing contract to a non-revealing contract. Here, the relevant comparison is therefore the revealing contract that does not induce investment.

The principal will therefore wish to induce investment as long as $V_R(p_1, 0) - (p_1 - p_0)i > V_R(p_0, 0)$. That is, she wishes to induce investments of a cost up to $v(\underline{q}) - v(\overline{q}) - \Delta q \underline{\theta}$. We summarize this in the following proposition:

**Proposition 47** *When the service provider owns the asset and contracts out the building of the asset, investment can be induced up to an investment cost of* $i_{PFI2} = v(\underline{q}) - v(\overline{q}) - \Delta q \underline{\theta}$.

**Proof.** The proposition follows from the argument in the text. ■

[9] The service provider cannot choose greater damage payments than stipulated by the principal's direct transfer since she needs to be able to ensure the builder's participation. Observability of the contract between service provider and builder makes this contractible by the principal.

### 5.4.3 Incentives to Invest

We can now ask the question of which regime provides better incentives for investment in quality improvement. We know that, when the principal owns the asset, investment can be induced (by judicious choice of the expected damage payments) up to investment costs of $i_{TRAD} = v(\underline{q}) - v(\overline{q}) - \Delta q \underline{\theta}$.

We also know that when the service provider sub-contracts the building of the asset, the principal wishes to make transfers to the service provider (to ensure the participation of the builder) up to the point where investments of cost $i_{PFI2} = v(\underline{q}) - v(\overline{q}) - \Delta q \underline{\theta}$ are undertaken. These two cases therefore yield the same levels of investment. Furthermore, it is straightforward to check that the principal's *ex ante* expected level of utility (taking into account that she will have to maintain the builder's participation constraint) is identical in both cases.

When the service provider owns and builds the asset, the principal can induce investment by writing the lowest-rent separating contract, for all investment costs below $\overline{q}\Delta\theta$. The principal also has an incentive to induce investment for higher investment costs, *viz.* by increasing subsidies (rent) to the service provider-builder; this induces investment for all investment costs such that (for the service provider-builder), the cost of investing is outweighed by increased rent. That is, by increasing the subsidy to the service provider-builder, the principal wishes to induce investment up to an investment cost of $i_{PFI1} = \frac{p_1 - p_0}{p_1}\left[v(\underline{q}) - v(\overline{q})\right] - \frac{p_1 - p_0}{p_1}\left[\underline{q}\underline{\theta} - \overline{q}\overline{\theta}\right] + \frac{p_0}{p_1}\overline{q}\Delta\theta$. For investment costs higher than this, the principal no longer wishes to induce investment and therefore writes the revealing (low-rent) contract.

We find that asset ownership and provision by the service provider may result in greater investment than traditional service provision. The following proposition contains necessary conditions under which this will be the case. First, we make the following definition:

**Definition 48** *A public service is* essential *when demand for the service is inelastic, that is if* $|\eta| \equiv -\frac{dq}{dq^{-1}}\frac{q^{-1}}{q} < 1$.

The definition expresses the (standard) presumption that a service is essential when demand is relatively unresponsive to price changes. We can now formulate our central proposition:

**Proposition 49** *When the public service is essential, asset ownership and provision by the service provider leads to greater investment incentives than traditional public service provision.*

**Proof.** We wish to find conditions such that

$$\frac{p_1 - p_0}{p_1}\left[v(\underline{q}) - v(\overline{q})\right] - \frac{p_1 - p_0}{p_1}\left[\underline{q}\underline{\theta} - \overline{q}\overline{\theta}\right] + \frac{p_0}{p_1}\overline{q}\Delta\theta > v(\underline{q}) - v(\overline{q}) - \Delta q\underline{\theta}.$$

Rewriting, we obtain

$$v(\underline{q}) - v(\overline{q}) < \Delta q\underline{\theta} + \frac{p_1}{p_0}\overline{q}\Delta\theta \tag{5.11}$$

and dividing by $\Delta q$, we have

$$\frac{v(\underline{q}) - v(\overline{q})}{\Delta q} < \underline{\theta} + \frac{p_1}{p_0}\overline{q}\frac{\Delta\theta}{\Delta q}. \tag{5.12}$$

Recall that $v(q) = \int_0^q q^{-1}(t)dt$. Since $\frac{dq^{-1}}{dq} < 0$, and $q^{-1}(\underline{q}) = \underline{\theta}$, we have

$$v(\underline{q}) - v(\overline{q}) = \int_{\overline{q}}^{\underline{q}} q^{-1}(t)dt \geq \int_{\overline{q}}^{\underline{q}} \underline{\theta}dt = \underline{q}\underline{\theta} - \overline{q}\underline{\theta} = \Delta q\underline{\theta},$$

and

$$v(\underline{q}) - v(\overline{q}) = \int_{\overline{q}}^{\underline{q}} q^{-1}(t)dt \leq \int_{\overline{q}}^{\underline{q}} \overline{\theta}dt = \underline{q}\overline{\theta} - \overline{q}\overline{\theta} = \Delta q\overline{\theta}.$$

We therefore need $\frac{p_1}{p_0}\overline{q}\frac{\Delta\theta}{\Delta q}$ sufficiently large. Denote $\tilde{\theta} \equiv \frac{v(\underline{q})-v(\overline{q})}{\Delta q}$, with $\underline{\theta} \leq \tilde{\theta} \leq \overline{\theta}$,[10] so that (5.12) rewrites as

$$\tilde{\theta} < \underline{\theta} + \frac{p_1}{p_0}\overline{q}\frac{\Delta\theta}{\Delta q}$$

[10] Note that, since $v(q) = \int_0^q q^{-1}(t)dt$, the fraction $\frac{v(\underline{q})-v(\overline{q})}{\Delta q}$ can be thought of as the average value of inverse demand over the interval $[\overline{q}, \underline{q}]$.

or

$$1 < \frac{\theta}{\overline{\theta}} + \frac{p_1}{p_0}\frac{\overline{q}}{\overline{\theta}}\frac{\Delta\theta}{\Delta q}.$$

The expression $\frac{\overline{q}}{\theta}\frac{\Delta\theta}{\Delta q}$ is approximated well by $\frac{1}{|\eta|}$, where $\eta$ is the elasticity of demand for the service. Since $\frac{\theta}{\overline{\theta}} < 1$, and $p_1 > p_0$, this inequality is satisfied if $|\eta| < 1$. This proves the proposition. ■

The following two examples illustrate this proposition.

**Example 50** *This example illustrates for a linear demand curve our result that production on the inelastic part of the demand curve always guarantees greater investment under private asset ownership. Let demand for the service output be linear, $q^{-1}(q) = a - bq$. Of course $\eta = 1 - \frac{a}{bq}$, and demand is elastic for all $q \in [0; \frac{a}{2b}]$, and inelastic for all $q \in [\frac{a}{2b}; \frac{a}{b}]$. Also, we have $v(q) = \int_0^q (a - bt)dt = aq - \frac{b}{2}q^2$. Substituting into (5.11) we obtain $\underline{q} < \left(1 + 2\frac{p_1}{p_0}\right)\overline{q}$. Suppose we are on the inelastic part of the demand curve, that is $\frac{a}{b} \geq \underline{q} > \overline{q} \geq \frac{a}{2b}$. Choose $\underline{q} > \overline{q}$ such that it is most difficult to satisfy (5.11),* viz. *$\underline{q} = \frac{a}{b}$, and $\overline{q} = \frac{a}{2b}$. In this case, (5.11) states that $\frac{a}{b} < \left(1 + 2\frac{p_1}{p_0}\right)\frac{a}{2b}$. By $p_1 > p_0$ this is always true.*

**Example 51** *This example is a counter-example: for a linear demand curve, when production takes place on the elastic part of the demand curve, private asset ownership may be investment-dominated by public asset ownership. Again, let $q^{-1}(q) = a - bq$. Suppose production occurs on the elastic part of the demand curve, so that $q \in [0; \frac{a}{2b}]$. It is simple to construct an example in which $\underline{q} < \left(1 + 2\frac{p_1}{p_0}\right)\overline{q}$ does not hold: let $\overline{q} = \varepsilon$, for $\varepsilon$ small. Since we stipulated $\underline{q} > \overline{q}$, (5.11) does not hold in this case.*

### 5.4.4 Choosing the Institutional Framework

The final question we need to answer is: when will the principal procure (and own) the asset, and when will she delegate ownership to a private investor (service provider)? Suppose that the assumptions of proposition 49 hold, so that private ownership induces higher levels of investment.

We know from propositions 44 and 47 that the principal is indifferent between traditional public service provision and private sector involvement when the service provider contracts out the building of the asset. We therefore focus on the choice between private sector involvement when the service provider builds the asset herself, and when the asset is procured by the principal (or procurement is contracted out by the service provider).

Consider first investments that can be induced both under traditional public service provision, and under PFI. For investments that can be induced both when the principal owns the asset and when the service provider owns the asset (that is, for all investment costs $i \leq v(\underline{q}) - v(\overline{q}) - \Delta q \underline{\theta}$, state ownership (when investment is induced) gives the principal an *ex ante* expected payoff of $V_R(p_1, 0) - (p_1 - p_0)i$. Asset ownership by the service provider gives the principal a payoff of $\tilde{V}_R(p_1, 0, \underline{s}^*) = p_1 \left[v(\underline{q}) - \underline{q}\underline{\theta} - \underline{s}^*\right] + (1 - p_1) \left[v(\overline{q}) - \overline{q}\overline{\theta}\right]$, where investment can be induced up to $i = \underline{s}^*$, so that the principal's payoff when investment $i$ is induced is $\tilde{V}_R(p_1, 0, i)$. A revealing contract that does not induce investment gives the principal an expected payoff of $V_R(p_0, 0)$. A non-revealing contract gives the principal a payoff of $V_H(0)$. From lemma 43 we know that $V_R(p_0, 0) \geq V_H(0)$, so that the non-revealing contract will never be chosen. But we also know that $\tilde{V}_R(p_1, 0, i) \geq V_R(p_0, 0)$ for the range of $i$ we are currently considering (i.e. $i \leq v(\underline{q}) - v(\overline{q}) - \Delta q \underline{\theta}$), so that asset ownership by the service provider (with subsidy payments sufficiently high to induce investment) is always preferred to any revealing contract that induces no investment.[11] So the choice of institutional framework for the ownership of assets necessary for public service production is just a choice between principal ownership (when it induces investment) and service-provider ownership (when it induces investment), when the required investment cost admits both structures as structures that could induce investment.

The principal should of course choose the structure that maximizes her *ex ante* expected payoff; that is, she should choose ownership by the principal when[12]

$$V_R(p_1, 0) - (p_1 - p_0)i \geq \tilde{V}_R(p_1, 0, \max\{i, \overline{q}\Delta\theta\}).$$

[11] Of course, when proposition 49 holds, $i \leq v(\underline{q}) - v(\overline{q}) - \Delta q \underline{\theta}$ implies, $\underline{s}^* = i < \frac{p_1 - p_0}{p_1} \left[v(\underline{q}) - v(\overline{q})\right] - \frac{p_1 - p_0}{p_1} \left[\underline{q}\underline{\theta} - \overline{q}\overline{\theta}\right] + \frac{p_0}{p_1} \overline{q} \Delta\theta$, which implies that $\tilde{V}_R(p_1, 0, \underline{s}^*) \geq V_R(p_0, 0)$.

[12] Recall that the subsidy to the low-cost service provider needs to be at least $\overline{q}\Delta\theta$ for revelation of cost conditions.

This of course is just

$$i \leq \frac{p_1}{p_1 - p_0} \max\{i, \overline{q}\Delta\theta\} - \frac{p_1}{p_1 - p_0}\overline{q}\Delta\theta.$$

For all $i < \overline{q}\Delta\theta$, the principal therefore prefers ownership by the service-provider. For all $i \geq \overline{q}\Delta\theta$, the principal prefers to own the asset herself if $i \geq \frac{p_1}{p_0}\overline{q}\Delta\theta$, and prefers ownership by the service provider if $i < \frac{p_1}{p_0}\overline{q}\Delta\theta$. The intuition for this result is simple: for very low values of the investment cost (up to $\frac{p_1}{p_0}\overline{q}\Delta\theta$), the principal prefers to induce the investment just through the rent payment to the service provider (which needs to be paid to the service provider anyhow in order to induce revelation). For $i \geq \frac{p_1}{p_0}\overline{q}\Delta\theta$, the principal prefers to own the asset herself: investment in asset quality is, to the principal, less costly: when the asset is privately owned, she can induce investment only by paying the service provider the investment cost as part of the subsidy when (observed) cost is low, that is, both in states 1 and 2. When the principal owns the asset, she only needs to reimburse the builder with the expected investment cost (that is, only when state 2 occurs).

For investments that can only be induced through private asset ownership, i.e. for all $i$ such that

$$v(\underline{q}) - v(\overline{q}) - \Delta q\underline{\theta} < i \leq \frac{p_1 - p_0}{p_1}\left[v(\underline{q}) - v(\overline{q})\right] - \frac{p_1 - p_0}{p_1}\left[\underline{q\theta} - \overline{q\theta}\right] + \frac{p_0}{p_1}\overline{q}\Delta\theta,$$

we know, by construction of $\underline{s}^* = \frac{p_1 - p_0}{p_1}\left[v(\underline{q}) - v(\overline{q})\right] - \frac{p_1 - p_0}{p_1}\left[\underline{q\theta} - \overline{q\theta}\right] + \frac{p_0}{p_1}\overline{q}\Delta\theta$ that the principal prefers the revealing contract that induces investments of this size to the revealing contract that does not induce investment. Any revealing contract that does not induce investment gives the principal the same *ex ante* expected payoff—whether the principal or the service provider owns the asset. Again, by lemma 43 we know that these revealing contracts are preferred to a non-revealing contract. It is therefore straightforward that for investment costs that can only be induced through private asset ownership, PFI as a method for public service provision will be chosen.

We summarize the preceding discussion in the following proposition:

**Proposition 52** *For all $i$ such that $i < \frac{p_1}{p_0}\overline{q}\Delta\theta$ and $i \leq v(\underline{q}) - v(\overline{q}) - \Delta q\underline{\theta}$, private (service provider) ownership of the asset is optimal for the principal. For all $i$ such that $i > \frac{p_1}{p_0}\overline{q}\Delta\theta$*

*and $i \leq v(\underline{q}) - v(\overline{q}) - \Delta q\underline{\theta}$, public (principal) ownership is optimal for the principal. For all $i > v(\underline{q}) - v(\overline{q}) - \Delta q\underline{\theta}$ but $i \leq \frac{p_1 - p_0}{p_1}\left[v(\underline{q}) - v(\overline{q})\right] - \frac{p_1 - p_0}{p_1}\left[\underline{q}\underline{\theta} - \overline{q}\overline{\theta}\right] + \frac{p_0}{p_1}\overline{q}\Delta\theta$, private (service provider) ownership of the asset is optimal for the principal.*

**Proof.** The proposition follows from the preceding discussion. ■

The proposition states that, as is intuitive, for small required investments, and for investments that improve asset quality dramatically ($\frac{p_1}{p_0}$ is large), private ownership is optimal. This is straightforward: for small investment costs (or very effective investments), government would rather provide investment incentives through the cheaper way of the rent paid to the service provider. This is less costly for the principal since the service provider needs to obtain rent anyway in order to reveal type to the principal. For small investment costs, this (standard) rent is sufficient for the service provider to want to invest in quality. If the principal were to procure the asset directly, she would need to pay the builder separately for the expected investment cost. For intermediate levels of cost (up to the point at which public ownership of the asset can no longer induce investment), public ownership is optimal. These levels of investment costs are those for which it is cheaper for government to provide investment incentives directly: government needs to reimburse the builder's expected cost (i.e. cost which arises in state 2 only), whereas the subsidy to the service provider is paid in all states where cost is high (i.e. state 1 and 2). Finally, when investment can no longer be induced through public ownership, the principal again prefers, by construction, private ownership.

## 5.5 Conclusion

We have shown that under relatively palatable assumptions about public service provision (services that are provided are in some sense essential), private asset ownership leads to greater incentives for investment in asset quality. However, it is important to note that this crucially depends on the nature of demand (and the support of the distribution of $\theta$, and with it, the size of $\underline{q}$ and $\overline{q}$): when demand for the service output is elastic, this result cannot in general be guaranteed. The reason for this greater investment incentive is the internalization of an externality: the principal (the state) can commit, through transmitting ownership and the building of an asset to a private service provider, to not deviating in its contract setting in such a way as to always extract damages from the builder.

Presumably, the institutional framework (that is, should assets be owned publicly or privately) is a choice variable for the principal. We therefore studied the principal's choice between different ownership structures. We obtained the intuitively appealing conclusion that the principal prefers private ownership of assets for either small or large levels of investment required to reduce service provision costs. This gives a fuller answer to the question: "What should the state buy?" In our view, what the state should buy, under the conditions just highlighted, are services, not assets.

# Chapter 6

# Retailer Sales Commission and the Quality of Advice

In this chapter, we study the impact of transfer payments paid to a common retailer on the quality of advice customers obtain from the retailer. In particular, we are interested in the question of whether such side transfers (for instance, sales commissions to retailers) may lead a retailer to change her sales advice. The question naturally arises from a consideration of the common agency nature of the problem: in the case of common sales agency, a single retailer sells similar products of different producers. But unlike the general theory reviewed in chapters 3 and 4, this chapter focuses not on the informational asymmetry between retailer (agent) and manufacturers (principals), but between customer and retailer: the retailer knows more about the products she sells than her customer. We present our model within the framework of retail sales of life assurance and similar savings products.

Focusing on the informational asymmetry between customer and retailer allows us to study the question of whether the possibility of different manufacturers offering different sales commissions influences the quality of the advice the retailer provides for her customers. In our framework, products are vertically differentiated according to their quality: everyone prefers higher to lower quality. Generally, one might also want to address the question of how common sales agency (and incentive payments to retailers) influences sales behavior in a model of horizontal product differentiation. In that framework, the question of asymmetric information between retailer and producer arises naturally: the retailer will generally have more information about the tastes of her customers over the horizontal product space, and producers may want to make use of that information for purposes of product development or targeting to particular consumers. In that case, an analysis similar to that of Mezzetti (1997) would apply.[1] Since we analyze a vertically differentiated product space, our interest is naturally in the asymmetry between customer and retailer, and the consequences for the quality of advice: the retailer should always recommend the highest quality product available.

The point this chapter makes is that, since sales commissions to retailers are observable (by law in the case of sales agency for life assurance savings products, or by inference from the retailer's observable sales effort), they act as potential signals of product quality. We demonstrate the existence of a separating equilibrium in which sales commissions are a perfect guide to product quality. In this equilibrium, sales commissions therefore convey information about unobservable quality: in equilibrium, customers will obtain the highest quality product.

## 6.1 Introduction

One of the cornerstones of modern economics is the competitive paradigm of price theory, in which a countable set of homogeneous goods, uniquely characterized by a vector of nonnegative prices, are traded amongst symmetrically informed, rational agents operating in a complete set of competitive markets. It can be shown that under fairly general conditions the allocation that results from such trading has desirable welfare properties. Furthermore, the only information that agents need to possess in this framework is information about prices. In particular, the model implies that there is no role for intermediaries in trade (for instance, retailers).

However, when information is distributed asymmetrically among agents, many of the appealing properties of markets disappear. Worse yet, we have no good understanding of a product space in which goods are differentiated. Under these circumstances, trade in general will also depend on non-price characteristics of goods. In fact, in markets in which consumers possess no, or only imperfect, information about product characteristics,

[1] That paper is reviewed in chapter 4.

goods are often sold through intermediaries that apparently narrow the informational gap between the trading agents. In this case, we should expect informed sellers to compete in side payments (for instance, sales commissions) to intermediaries, rather than to compete purely in prices.

One way of understanding the economic role played by side payments such as sales commissions is as a way of conveying information about imperfectly known characteristics of goods. A simple intuition is the following: Suppose goods are differentiated along a quality dimension, and that quality is unobservable by consumers at the time of purchase. In this case, one might conceptualize commission payments from a perfectly informed seller to the retailer as a potential signal (in the sense of Spence (1974)) of product quality. This is the approach this paper explores.

One market in which intermediation is common is the market for financial services. Indeed, in the UK, commission payments to retailers of financial services have recently received much attention both in the press and from regulatory agencies. We therefore take this market as our motivating example.

A different way of viewing side payments is to focus on the informational asymmetry between sellers and retailer: the retailer has information about customers (or more generally, about market conditions) that sellers do not possess. Side payments (such as sales commissions) may be a way of eliciting that information. If a retailer acts on behalf of more than one seller, we have a typical common agency situation with asymmetric information (cf. Martimort (1992), Stole (1992)). As mentioned in the introduction to this chapter, such an approach applies to a horizontally differentiated product space. Here, we study the case of vertically differentiated products, where the limited information of producers about their customers does not naturally force the modelling as one of common agency with asymmetrically informed principals.

The remainder of this section introduces the general intuition and motivates our model by briefly describing the market for life assurance savings products in the UK. Section 6.2 introduces a general way of thinking about sales commission as signals of quality. Section 6.3 analyses three special cases of the general model. The final section 6.4 concludes with remarks on welfare implications.

### 6.1.1 Sales Commission and the Quality of Advice

Most financial services (for instance, life assurance savings products) in the UK are retailed through independent financial advisers (IFAs) who offer their customers advice on a portfolio of such policies. Usually, financial advisers receive commission payments from insurers, and such commission payments differ widely across apparently similar products. Despite the mandatory disclosure of commission payments to customers, these side payments have often been viewed as biasing the retailer's advice, and to lead consumers to buy inappropriate products.

In this paper, we interpret commission payments as a possible signal of unobserved product quality. The idea is that if high-quality products always earn higher commission payments than products of inferior quality, a retailer purely interested in maximizing commission revenue will always sell the appropriate policy.

In this sense, commission payments are akin to uninformative, but observable, advertising expenditure. The link between advertising for experience goods and product quality has been studied before (e.g. Nelson (1970), Nelson (1974), Kihlstrom and Riordan (1984), Milgrom and Roberts (1986)).[2] We know from this literature that quality signalling for experience goods cannot arise in a one-period model; subsequent periods, in which buyers acquire information about product quality are needed to achieve a separating equilibrium. A very rough intuition, of course, is that in order to achieve a separating equilibrium, the marginal return to advertising of a high-quality producer has to be greater than that of a low-quality producer. Unsurprisingly, the crucial condition turns out to be an "informativeness" condition: a separating equilibrium (one in which commission payments signal product quality) can exist only if observable characteristics of the product are somehow informative (in a sense to be made precise below) of product quality.

### 6.1.2 Commission Disclosure

The main conclusion of the paper is that, if a separating equilibrium exists, commission is an accurate guide to product quality. In this sense, the paper contributes to the policy

[2]Nelson (1970) distinguishes between experience goods (for which quality is not directly observable on inspection) and search goods (for which quality is verifiable on inspection). Clearly, directly informative advertising can only arise in equilibrium for search goods; statements about the quality of an experience good (since unverifiable on inspection) will rationally be ignored by consumers.

discussion on commission disclosure. In the UK, mandatory commission disclosure for life assurance savings products was introduced in 1995, and as a result, data on commission payments for these products is now readily available. We therefore choose to motivate many of the points made in the paper by referring to the retail market for life assurance savings products. The following subsection (6.1.3) briefly describes this market.

### 6.1.3 The Market for Financial Services in the UK

#### IFAs and Commission Dispersion

Since the 1986 Financial Services Act, independent financial advisers (IFAs) have become an important sales channel for financial products. A large, and growing, proportion of insurance savings products are now retailed through IFAs. Figures 6.1 and 6.2 show the share of new premiums sold through the various sales channels for the period 1993–1997. All figures are in the appendix to this chapter.

Around two thirds of IFAs operate purely on commission terms. Figures 6.3 and 6.4 are examples of the dispersion of commission payments; they present the total commission payment made to IFAs over the first 5 years of two different policies.[3] What these figures illustrate is the extent to which commission payments differ across insurers—*prima facie* evidence of the incentive to give biased advice.

#### Regulation

Savings products in the UK are currently regulated by the Personal Investment Authority (PIA). PIA introduced full commission disclosure rules on January 1, 1995. Disclosure rules for non-life products (unit trusts, investment trust savings schemes, etc.) were implemented on May 1, 1997. Despite full disclosure, however, commission payments continue to be perceived as biasing advice and leading to the mis-selling of financial products.

In this connection, our paper makes two points: First, if a separating equilibrium exists (in which commission payments signal product quality accurately), mandatory commission disclosure amounts to publishing product quality. Secondly, in this case, financial advisers (modelled here, admittedly simplistically, as maximizing only commission income) fulfil

[3] Figures are based on a gross premium of £60 per month.

only the role of allowing insurers to signal through commission payments. Clearly, this implies that if insurers could find alternative ways of publicly throwing away money (for instance, running expensive advertising campaigns), there is no need for financial advisers. This, clearly, is the less robust conclusion of the paper, and should therefore not be overemphasized. In fact, financial advisers may serve different economic functions, which we do not address here.[4]

### Persistency and Early Surrender

A final fact about the market for life assurance savings products that we might want to explain is that of higher persistency for products sold through IFAs. The persistency rate is the percentage of initial buyers of a policy that have not surrendered the policy before the end of its regular lifetime. Generally, early surrender (that is, low persistency) is more common for products sold through tied advisers than for those sold through IFAs. Figures 6.5 and 6.6 show persistency rates for policies sold through IFAs and tied advisers (company representatives and appointed representatives), respectively. This fact, of itself, should raise doubts about whether commission payments indeed are distorting the quality of sales advice, at least relative to the advice given by tied advisers.

## 6.2 The General Model

In our model, there are three time periods $t = 0, 1, 2$. We model the financial asset as a stochastic process with random per-period returns $X_t$, and denote realizations of each random variable by $x_t$.[5] In a very general formulation of the problem, agents observe some $y_t = y_t(x_t)$. Write $Y_t$ for $y_t(X_t)$. There is a seller of the financial asset and a buyer. Let the buyer's outside investment opportunities $r$ (rates of return) be distributed according to the distribution function $G(\cdot)$. If the buyer purchases the financial asset at time $t$, her period $t$ payoff is $\frac{x_t}{p} - 1$. Payoffs are discounted at rate $\delta$, $0 < \delta \leq 1$. The seller can signal (pay a commission) $s_0$ per unit sold in period $t = 0$. Denote the period $t$ demand for the asset by

[4] For instance, they may acquire information about customers when products are horizontally differentiated. We discuss this in some more detail below.

[5] Generally, we use capitals to denote random variables, and lower-case letters for their realizations.

$q_t$, and its price by $p$.[6] The seller's expected profit therefore is $(p - s_0)q_0 + \delta p E[q_1 | Y_0 = y_0]$. The retailer's objective function is common knowledge, so that for some specification of her objective (for instance, she maximizes commission income) or her behavior (say, she apportions the time spent explaining each policy according to the commission payments she receives on each), consumers can induce the amount of commission payments. We can therefore ignore the role of the retailer in the following modelling.

The timing of the model is as follows:

- at $t = 0$, $x_0$ realizes; the seller observes $y_0$ and signals $s_0(\hat{y}_0)$; the buyer infers product quality $\tau^{-1}(s_0)$, where $\tau : \Upsilon \rightarrow \Xi$ ($\Upsilon$ is the set of possible values of $y_0$ and $\Xi$ is the set of all possible values of $x_0$) and decides whether to buy the asset; if the buyer does not buy the asset, the game ends with zero payoffs for both players;

- at $t = 1$, $x_1$ realizes; the buyer observes $y_1$ and decides whether to repeat purchase of the asset; if she decides not to repeat purchase, the game ends and payoffs are realized; since we wish to model repeat purchase behavior, at $t = 1$, the asset can only be purchased by period $t = 0$ buyers;

- at $t = 2$, $x_2$ realizes; the game ends, and payoffs are realized.

### 6.2.1 The Buyer's Problem

At time $t = 0$, the buyer purchases the asset if, and only if, the expected rate of return from holding the asset (taking into account that only buying the asset now gives the option of repeating purchase at $t = 1$) is greater than the buyer's outside rate of return. Specifically, the buyer purchases the asset at $t = 0$ if

$$\begin{aligned} & (1+\delta)r \\ \leq \; & E_{X_1|Y_0}\left[\frac{X_1}{p} - 1 + \delta \max\left\{\frac{E_{X_2|Y_1,Y_0}\left[X_2|Y_1, s_0(Y_0) = s_0(\hat{y}_0)\right]}{p} - 1, r\right\} |s_0(Y_0) = s_0(\hat{y}_0)\right]. \end{aligned}$$

[6]Note that in this model, price is not a choice variable. This is justified by the fact that, for life assurance savings products, the termly premium is typically chosen by the buyer. If sellers could choose prices as well as commission payments, one might expect signalling to occur through prices as well as sales commissions.

That is, demand at $t = 0$ is

$$q_0 = \Pr\left\{(1+\delta)r \le E_{X_1|Y_0}\left[\frac{X_1}{p} - 1 + \right.\right. \tag{6.1}$$
$$\left.\left. +\delta \max\left\{\frac{E_{X_2|Y_1,Y_0}\left[X_2|Y_1, s_0(Y_0) = s_0(\hat{y}_0)\right]}{p} - 1, r\right\} |s_0(Y_0) = s_0(\hat{y}_0)\right] |Y_0 = y_0\right\}.$$

Given that the buyer has purchased at $t = 0$, she repeats purchase at $t = 1$ if, and only if,

$$r \le \frac{E_{X_2|Y_1,Y_0}\left[X_2|Y_1 = y_1, s_0(Y_0) = s_0(\hat{y}_0)\right]}{p} - 1,$$

so that period 1 demand is

$$q_1 = \min\left\{q_0, \Pr\left\{r \le \frac{E_{X_2|Y_1,Y_0}\left[X_2|Y_1 = y_1, s_0(Y_0) = s_0(\hat{y}_0)\right]}{p} - 1\, |Y_0 = y_0\right\}\right\}. \tag{6.2}$$

### 6.2.2 The Seller's Profit

The seller's expected profit, after observing $y_0$, is consequently

$$(p - s_0(\hat{y}_0))\, q_0 + \delta p E_{Y_1|Y_0}\left[q_1|Y_0 = y_0\right],$$

where, of course, the conditional expectation in $q_1$ is conditional on $Y_1 = y_1|_{y_1=Y_1}$, i.e. evaluated at the random variable $Y_1$.

### 6.2.3 Three Models

To demonstrate the generality of the notation, consider the following three models:

#### Model 1

Let each $X_t$ be distributed according to the same one-parameter distribution with parameter $\theta \in \Theta$, so that $X_t = X$. Let this parameter be observable to sellers at period 0 and buyers at period 1, so that $y_t = y_t(\cdot) = \theta$ (and denote $\hat{y}_t = \hat{\theta}$). Interpret $\theta$ as the quality of the asset (initially only observable to sellers). This is the simplest model of vertical product differentiation: the asset has a certain (fixed) quality, and realizations of the asset are drawn according to a distribution function with parameter $\theta$.

### Model 2

Let agents observe the realization of the $X_t$, so that $y_t = y_t(x_t) = x_t$ (and denote $\hat{y}_t = \hat{x}_t$). Furthermore, let the $X_t$ be independently distributed. In this model, there is no fixed quality of the asset. Higher realizations are better than lower realizations, but each per-period rate of return on the asset is independently drawn.

### Model 3

As in model 2, let agents observe the realization of the $X_t$, so that $y_t = y_t(x_t) = x_t$ (and denote $\hat{y}_t = \hat{x}_t$). However, let the $X_t$ follow a stochastic process $\{X_t\}$ that is Markovian. In this model, asset quality evolves over time: this period's payoff is the mean to next period's payoff. If an asset starts off with a high return ($x_0$), then its quality (that is, its likely future returns) is high. Conversely, a low-quality asset is one with a low realization $x_0$.

## 6.3 Analysis of Models 1–3

In order to establish the existence of a separating equilibrium, it is sufficient to check whether the seller's expected profit obeys a Spence-Mirrlees single-crossing condition. The nonlinearity of the general model however precludes a simple solution. We therefore focus on the case where $y_0$ is drawn from a set $\Upsilon$ with $\#\Upsilon = 2$. In this case, we need to show that there exists an equilibrium in which no seller can gain from "lying" about her private information. (This of course, is the two-type equivalent of checking for single-crossing.)

### 6.3.1 Model 1

In this model, each $X_t = X$ is distributed according to the same one-parameter distribution with parameter $\theta$. This parameter is observable to sellers at period 0 and buyers at period 1, so that $y_t = y_t(\cdot) = \theta$. The model is thus one in which a financial product of a certain quality $\theta$ is sold, where $\theta \in \Theta$. Refer to the seller of this product as "firm $\theta$". Before $\theta$ is observed, refer to it as the random variable $\tilde{\theta}$. Firm $\theta$ produces an asset with random per-period return $X$, distributed according to the cumulative distribution function $F(x|\theta)$

with conditional density $f(x|\theta) > 0$ on $[\underline{x}, \overline{x}]$. In period 0, $\theta$ is the firm's private information. Once first-period buyers have experienced the product, they know its quality perfectly; in period 1, therefore, $\theta$ is known to all players (in fact, it is sufficient to assume that it is known to all first-period buyers).

In this case (6.1) reduces to

$$q_0 = \Pr\left\{(1+\delta)r \leq E_{X|\tilde{\theta}}\left[\frac{X}{p} - 1 + \delta \max\left\{\frac{E_{X|\tilde{\theta}}\left[X|\tilde{\theta} = \hat{\theta}\right]}{p} - 1, r\right\} \middle| \tilde{\theta} = \hat{\theta}\right]\right\}.$$

Note that this further reduces to[7]

$$q_0 = \Pr\left\{r \leq \frac{E[X|\tilde{\theta} = \hat{\theta}]}{p} - 1\right\}.$$

Furthermore, (6.2) can be rewritten

$$q_1 = \min\left\{q_0, \Pr\left\{r \leq \frac{E_{X|\tilde{\theta}}\left[X|\tilde{\theta}\right]}{p} - 1 \middle| \tilde{\theta} = \theta\right\}\right\}.$$

For concreteness, denote the asset's expected return conditional on $\theta$ by $E[X|\theta] = \int_{\underline{x}}^{\overline{x}} x dF(x|\theta)$.[8] We make the assumption that, for any $\theta' > \theta$, $F(x|\theta')$ stochastically dominates $F(x|\theta)$.[9] For generality, let firm $\theta$'s unit production cost be $c_\theta$, and let the asset's price be exogenously fixed at $p$.[10] Further, assume that $c_\theta < p$ for all $\theta$. (If, to the contrary, for some firm $c_\theta \geq p$, that firm would never make positive profits, and would therefore never enter the market.) Each firm $\theta$ can pay the retailer a commission $s_0$ (which, in general, will be a function $s_0 : \Theta \rightarrow S$, where $S = \{s | s \in \Re, s \geq 0\}$). Customers draw an inference $\hat{\theta} = \tau^{-1}(s_0)$ about firm type, where $\tau : \Theta \rightarrow S$. Note that in a separating equilibrium,

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[7]For assume that $r > \frac{E_{X|\tilde{\theta}}[X|\tilde{\theta}=\hat{\theta}]}{p} - 1$. Then we have $q_0 = \Pr\left\{(1+\delta)r \leq E_{X|\tilde{\theta}}\left[\frac{X}{p} - 1 + \delta r \middle| \tilde{\theta} = \hat{\theta}\right]\right\}$, or $q_0 = \Pr\left\{r \leq E_{X|\tilde{\theta}}\left[\frac{X}{p} - 1 \middle| \tilde{\theta} = \hat{\theta}\right]\right\}$ which is zero, and the case is not of analytical interest.

[8]If customers are risk averse, with utility function $u(\cdot)$, $u' > 0$, $u'' < 0$, then the interest is in the expected utility; because of the assumption of stochastic dominance, which we will make shortly, this has no effect on our model. However, stochastic dominance may be felt to be too strong an assumption, so that an investigation of risk-aversion may hold independent interest.

[9]Recall that the definition of first-order stochastic dominance is that, for any increasing function $h$, $\int_{\underline{x}}^{\overline{x}} h(x)dF(x|\theta') > \int_{\underline{x}}^{\overline{x}} h(x)dF(x|\theta)$.

[10]Note that in models 2 and 3 we do not introduce a cost-parameter since it is no longer clear how cost varies with "quality".

$\tau(\cdot) = s_0(\cdot)$. (Here, we are intentionally more explicit about buyer inferences than in the general model.)

Let $G(\cdot)$, the distribution of buyer outside investment opportunities, be uniform on $[0, 1]$. Suppose that, in each period, any one customer has use for at most one policy. Accordingly, if $\theta$ were known, customers $r \leq \frac{E[x|\theta]}{p} - 1$ would want to buy the asset. By assumption, however, the asset can only be bought in period 1 (i.e. the policy is renewed) if it was bought in period 0.

### Analysis of Model 1

It turns out to be convenient to rewrite demand for the asset slightly. In period 0, a customer will buy a policy she is offered if its expected return (which depends on her belief about the policy's quality) is greater than her outside investment opportunity. (Note that, in this model, buyers are willing to buy in period 0 only if they are (*ex ante*) also willing to buy in period 1.) With a uniform distribution of outside investment opportunities, therefore, period 0 demand for a product of (unknown) quality $\theta$, when customers draw inference $\hat{\theta} = \tau^{-1}(s_0)$ about type, is therefore

$$q_0(s_0) = \max\left\{0, \frac{E[X|\tau^{-1}(s_0)]}{p} - 1\right\}.$$

Since only period-one buyers can renew their policies in period 2, the period 2 demand is

$$q_1(s_0, \theta) = \min\left\{\max\left\{0, \frac{E[X|\tau^{-1}(s_0)]}{p} - 1\right\}, \max\left\{0, \frac{E[X|\theta]}{p} - 1\right\}\right\}.$$

Profits of a $\theta$-firm that signals $s_0$ are:

$$(p - c_\theta - s_0)q_0(s_0) + \delta(p - c_\theta)q_1(q_0, \theta).$$

We have:

$$\pi(s_0, q_0, \theta) = (p - c_\theta - s_0)\max\left\{0, \frac{E[X|\tau^{-1}(s_0)]}{p} - 1\right\} +$$

$$+\delta(p-c_\theta)\min\left\{\max\left\{0,\frac{E[X|\tau^{-1}(s_0)]}{p}-1\right\},\max\left\{0,\frac{E[X|\theta]}{p}-1\right\}\right\}$$

**The two-quality case**

To simplify the analysis, let $\Theta \equiv \{L,H\}$, $L<H$. We immediately have the following:

**Proposition 53** *There exists a separating equilibrium $s_0^*$, if and only if $p \le E[X|H]$ and*

$$(1+\delta)c_H \le c_L + \delta p. \tag{6.3}$$

**Proof.** First, observe that if $p > E[X|H]$, demand is zero for both firms. Therefore, $p \le E[X|H]$.

Observe first that, in a separating equilibrium, the low-quality firm's best choice of commission is 0. Therefore, in a separating equilibrium, we want $\pi(s_0^*, q_0(s_0^*); H) \ge \pi(0, q_0(0); H)$ and $\pi(0, q_1(0); L) \ge \pi(s_0^*, q(s_0^*); L)$, or:

$$\begin{aligned}
&\max\left\{0,\frac{E[X|H]}{p}-1\right\}(p-c_H-s_0^*)+\\
&+\delta\min\left\{\max\left\{0,\frac{E[X|H]}{p}-1\right\},\max\left\{0,\frac{E[X|H]}{p}-1\right\}\right\}(p-c_H)\\
\ge\ &\max\left\{0,\frac{E[X|L]}{p}-1\right\}(p-c_H)+\\
&+\delta\min\left\{\max\left\{0,\frac{E[X|L]}{p}-1\right\},\max\left\{0,\frac{E[X|H]}{p}-1\right\}\right\}(p-c_H)
\end{aligned}$$

and

$$\begin{aligned}
&\max\left\{0,\frac{E[X|L]}{p}-1\right\}(p-c_L)+\\
&+\delta\min\left\{\max\left\{0,\frac{E[X|L]}{p}-1\right\},\max\left\{0,\frac{E[X|L]}{p}-1\right\}\right\}(p-c_L)\\
\ge\ &\max\left\{0,\frac{E[X|H]}{p}-1\right\}(p-c_L-s_0^*)+\\
&+\delta\min\left\{\max\left\{0,\frac{E[X|H]}{p}-1\right\},\max\left\{0,\frac{E[X|L]}{p}-1\right\}\right\}(p-c_L).
\end{aligned}$$

Assuming that $p \leq E[X|L]$, these conditions reduce to:

$$s_0^* \leq (1+\delta)\left(\frac{E[X|H] - E[X|L]}{E[X|H] - p}\right)(p - c_H) \tag{6.4}$$

and

$$s_0^* \geq \left(\frac{E[X|H] - E[X|L]}{E[X|H] - p}\right)(p - c_L). \tag{6.5}$$

In this case, therefore, a separating equilibrium exists if and only if

$$(1+\delta)\left(\frac{E[X|H] - E[X|L]}{E[X|H] - p}\right)(p - c_H) \geq \left(\frac{E[X|H] - E[X|L]}{E[X|H] - p}\right)(p - c_L)$$

or

$$c_H \leq p + \frac{1}{1+\delta}(c_L - p)$$

or

$$(1+\delta)c_H \leq c_L + \delta p. \tag{6.6}$$

Assuming that $E[X|L] \leq p \leq E[X|H]$, the conditions reduce to:

$$s_0^* \leq (1+\delta)(p - c_H) \tag{6.7}$$

and

$$s_0^* \geq p - c_L. \tag{6.8}$$

In this case, a separating equilibrium exists if and only if

$$(1+\delta)c_H \leq c_L + \delta p \tag{6.9}$$

which is just the same condition as (6.6) above. ■

Note that conditions (6.6) and (6.9) can be rewritten, somewhat more intuitively, as

$$(1+\delta)p-(1+\delta)c_H \geq p-c_L$$

which is just the aforementioned inequality between the marginal return to signalling for high and low quality producers: at the margin, a high quality producer sells an additional unit in both periods (and bears the production cost in both periods); similarly, at the margin, a low quality producer only manages to sell an additional unit in the first period (bearing the production cost in that period), and sells nothing in the second period. For a separating equilibrium to exist, the former has to be greater than the latter.

In fact, we can establish whether the $H$-firm would want to advertise (that is, pay commission to its sales agents) to distinguish itself. This gives us:

**Corollary 54** *If, in addition to condition (6.3), we have the more stringent condition*

$$(1+\delta)c_H \leq c_L+\delta p+(c_L-p),$$

*the $H$-firm prefers to advertise to distinguish itself from the $L$-firm.*

**Proof.** Assume that, without advertising, customers' expectations about the policy's performance are $0.5E[X|L]+0.5E[X|H]$. In the resulting pooling equilibrium, a $\theta$ firm's profit is

$$\pi(0,q^P(0),\theta)=\max\left\{0,\frac{0.5E[X|L]+0.5E[X|H]}{p}-1\right\}(p-c_\theta)+$$

$$+\delta\min\left\{\max\left\{0,\frac{0.5E[X|L]+0.5E[X|H]}{p}-1\right\},\max\left\{0,\frac{E[X|H]}{p}-1\right\}\right\}(p-c_\theta) \quad (6.10)$$

An $H$-firm signalling at level $s_0^*$ in a separating equilibrium makes profits

$$\pi(s_0^*,q(s_0^*);H)=(1+\delta)\left(\frac{E[X|H]}{p}-1\right)(p-c_H)-\left(\frac{E[X|H]}{p}-1\right)s_0^*. \quad (6.11)$$

Note first from (6.10) that if $p \geq 0.5E[X|L]+0.5E[X|H]$, demand will be zero. In this case it is immediate from (6.11) that an $H$ firm will prefer to advertise if the advertising expenditure required to distinguish itself is

$$s_0^* \leq (1+\delta)(p - c_H),$$

which is just the same as condition (6.7), and weaker than (6.4). Therefore, assume in what follows that $p < 0.5E[X|L] + 0.5E[X|H]$.

In a (no-advertising) pooling equilibrium, in which customers' prior on qualities is $p_L = p_H = 0.5$, an $H$-firm's profit is

$$\pi(0, q^P(0); H) = (1+\delta)\left(\frac{E[X|H] + E[X|L]}{2p} - 1\right)(p - c_H). \tag{6.12}$$

For the $H$-firm to prefer to advertise, we need (6.11) > (6.12), so

$$s_0^* \leq \frac{1}{2}(1+\delta)(p - c_H)\frac{E[X|H] + E[X|L]}{E[X|H] - p}. \tag{6.13}$$

Suppose first that $p \leq E[X|L]$. For (6.13) to be possible in a separating equilibrium, we need (from condition (6.5))

$$\frac{1}{2}(1+\delta)(p - c_H)\frac{E[X|H] + E[X|L]}{E[X|H] - p} \geq \frac{E[X|H] - E[X|L]}{E[X|H] - p}(p - c_L)$$

or

$$c_H \leq p + 2\frac{1}{1+\delta}(c_L - p)$$

or

$$(1+\delta)c_H \leq c_L + \delta p + (c_L - p), \tag{6.14}$$

which is stronger than condition (6.3).

Now suppose $p > E[X|L]$. For (6.13) to be possible in a separating equilibrium, we need (from condition (6.8))

$$0.5(1+\delta)(p - c_H)\frac{E[X|H] + E[X|L]}{E[X|H] - p} \geq p - c_L.$$

or

$$c_H \leq p - \frac{2(p - c_L)}{(1+\delta)\frac{E[X|H]+E[X|L]}{E[X|H]-p}}$$

However, we know (from $p > E[X|L]$) that

$$\frac{E[X|H] + E[X|L]}{E[X|H] - p} > \frac{E[X|H] + E[X|L]}{E[X|H] - E[X|L]},$$

so that we have as a necessary condition for (6.13)

$$c_H \leq p + 2\frac{E[X|H] - E[X|L]}{E[X|H] + E[X|L]} \frac{1}{(1+\delta)}(c_L - p),$$

which is stronger than condition (6.3) if $\frac{E[X|H]-E[X|L]}{E[X|H]+E[X|L]} > 0.5$, but always weaker than (6.14).

■

Of course, the $L$-type firm would never prefer signalling to not signalling.

### 6.3.2 Model 2

In this model, agents observe the realization of the $X_t$, so that $y_t = y_t(x_t) = x_t$ (and denote $\hat{y}_t = \hat{x}_t$). Furthermore, let the $X_t$ be independently distributed. The model is therefore one in which the seller observes how well the asset has performed, and then signals regarding that information.

Note that if the $X_t$ are independent, period-0 demand is

$$q_0 = \Pr\left\{(1+\delta)r \leq E\left[\frac{X_1}{p} - 1 + \delta \max\left\{\frac{E[X_2]}{p} - 1, r\right\}\right]\right\}.$$

Furthermore, if the $X_t$ are independent, we have

$$q_1 = \min\left\{q_0, \Pr\left\{r \leq \frac{E[X_2]}{p} - 1\right\}\right\}.$$

Finally note that, if the $X_t$ are independent, the seller's profit does not depend on $x_0$. We therefore have the following:

**Proposition 55** *If per-period returns are independent, signalling cannot occur.*

**Proof.** The proposition follows from the derivation above. ■

The proposition is intuitive: if the buyer of an asset never learns whether the seller has lied to her about the asset's quality, there can be no separating equilibrium (there is no difference in marginal returns to signalling for sellers who observe different performance of the asset).

### 6.3.3 Model 3

We have just shown that if asset returns are independent, no signalling can occur. Signalling "works" in model 1 because per-period returns are identically distributed, and the shape of the distribution function is learned after the initial purchase. Model 1, however, is unsatisfactory for different reasons: it seems more natural to think of asset returns as following, for instance, a random walk. Model 3 therefore makes a "Markovian" assumption about $\{X_t\}$. As in model 2, agents observe the realization of the $X_t$, so that $y_t = y_t(x_t) = x_t$ (and denote $\hat{y}_t = \hat{x}_t$). However, let the $X_t$ follow a stochastic process $\{X_t\}$ that is Markovian, i.e. we have $E[X_t|X_{t-1}, X_{t-2}, ...] = E[X_t|X_{t-1}]$.

When $\{X_t\}$ is Markovian, (6.1) reduces to:

$$q_0 = \Pr\left\{(1+\delta)r \le E_{X_1|X_0}\left[\frac{X_1}{p} - 1 + \delta\max\left\{\frac{E_{X_2|X_1}[X_2|X_1]}{p} - 1, r\right\}|s_0(X_0) = s_0(\hat{x}_0)\right]\right\}$$

and (6.2) turns into

$$q_1 = \min\left\{q_0, \Pr\left\{r \le \frac{E_{X_2|X_1}[X_2|X_1]}{p} - 1\,|X_0 = x_0\right\}\right\}.$$

In this case, $q_0$ depends only on $\hat{x}_0$; $E_{X_1|X_0}[q_1|X_0 = x_0]$, however, does depend on $x_0$. There is, therefore, scope for the existence of a separating equilibrium.

#### Analysis of Model 3

In fact, let us restrict further the nature of the stochastic process and assume that $\{X_t\}$ is a martingale. Note, for later use, that we have the following basic property from martingale theory: $E[X_n|X_0, ..., X_k] = X_k$, for $k < n$. Adopt similar simplifications as in model 1 (but note that now "quality" refers to observed performance in period 0, i.e. $x_0 \in [\underline{x}_0, \overline{x}_0]$). Use the notation introduced for model 1, but let $\tau : [\underline{x}_0, \overline{x}_0] \to S$. Then, in period 1, a buyer with outside investment opportunity $r$ will buy (provided she has bought in period 0) if

$$\frac{E[X_2|X_1 = x_1, \tau^{-1}(s_0)]}{p} - 1 = \frac{x_1}{p} - 1 \geq r.$$

In period 0, this buyer will buy if: either

$$\frac{E[X_1|\tau^{-1}(s_0)]}{p} - 1 = \frac{\tau^{-1}(s_0)}{i} - 1 \geq r \tag{6.15}$$

or

$$\left(\frac{E[X_1|\tau^{-1}(s_0)]}{p} - 1\right) + \delta\left(\frac{E\left[E[X_2|X_1]\,|\tau^{-1}(s_0)\right]}{p} - 1\right) \geq (1+\delta)r$$

which, using the martingale property, can be rewritten as

$$\left(\frac{E[X_1|\tau^{-1}(s_0)]}{p} - 1\right) + \delta\left(\frac{E[X_1|\tau^{-1}(s_0)]}{p} - 1\right) \geq (1+\delta)r,$$

or

$$\left(\frac{\tau^{-1}(s_0)}{p} - 1\right) \geq r,$$

which is just the same as (6.15).

Writing down demands, we have

$$q_0 = \max\left\{0, \frac{\tau^{-1}(s_0)}{p} - 1\right\}, \tag{6.16}$$

and

$$q_1 = \min\left\{q_0, \max\left\{0, \frac{X_1}{p} - 1\right\}\right\}. \tag{6.17}$$

The seller's expected profit is

$$(p - s_0)\max\left\{0, \frac{\tau^{-1}(s_0)}{p} - 1\right\} + \delta p E_{X_1|X_0} \min\left\{q_0, \max\left\{0, \frac{X_1}{p} - 1\right\}\right\}.$$

At this point we need to simplify drastically in order to get a solution. Let all possible rates of return of the asset be contained within the unit interval. The seller's expected profit is then

$$(p - s_0)\left[\frac{\tau^{-1}(s_0)}{p} - 1\right] + \delta p E_{X_1|X_0} \min\left\{q_0, \left[\frac{X_1}{p} - 1\right]\right\}$$

Similarly to the two-quality case of model 1, let $x_0 \in \{l, h\}$, with $h > l$, and accordingly modify the domain of $\tau$ so that $\tau : \{l, h\} \to S$. Further, let $x_1 \in \{L, M, H\}$, with $H > h > M > l > L$, and with the "transition probabilities" $p_{x_0 x_1}$, such that $\sum_{x_1 \in \{L,M,H\}} p_{x_0 x_1} = 1$. The martingale property requires that $p_{hH}H + p_{hM}M + p_{hL}L = h$ and $p_{lL}H + p_{lM}M + p_{lL}L = l$. Further define $\Delta_H \equiv H - h$, and $\Delta_L \equiv l - L$. In analogy to the analysis of model 1, we require, for a separating equilibrium to exist, the following version of the Spence-Mirrlees single-crossing property to hold:

$$\begin{aligned}
& (p - s_0)\left(\frac{h}{p} - 1\right) + \delta p\left(\frac{p_{hH}h + p_{hM}M + p_{hL}L}{p} - 1\right) \\
\geq \;& p\left(\frac{l}{p} - 1\right) + \delta p\left(\frac{(p_{hH} + p_{hM})l + p_{hL}L}{p} - 1\right),
\end{aligned}$$

that is

$$s_0 \leq \frac{(h - l) + \delta\left(p_{hH}h + p_{hM}M - (p_{hH} + p_{hM})l\right)}{\frac{h}{p} - 1}$$

and

$$\begin{aligned}
& p\left(\frac{l}{p} - 1\right) + \delta p\left(\frac{(p_{lH} + p_{lM})l + p_{lL}L}{p} - 1\right) \\
\geq \;& (p - s_0)\left(\frac{h}{p} - 1\right) + \delta p\left(\frac{p_{lH}h + p_{lM}M + p_{lL}L}{p} - 1\right),
\end{aligned}$$

that is

$$s_0 \geq \frac{(h - l) + \delta\left(p_{lH}h + p_{lM}M - (p_{lH} + p_{lM})l\right)}{\frac{h}{p} - 1}.$$

So a separating equilibrium exists if

$$p_{hH}h + p_{hM}M - (p_{hH} + p_{hM})l \geq p_{lH}h + p_{lM}M - (p_{lH} + p_{lM})l$$

Rewrite this using the definitions of $\Delta_H$ and $\Delta_L$ and the martingale property as

$$-p_{hH}\Delta_H + p_{hL}\Delta_L + h \geq -p_{lH}\Delta_H + p_{lL}\Delta_L + l.$$

We can now state our next result:

**Proposition 56** *In the martingale case of model 3, a separating equilibrium exists if $h-l$ is sufficiently large, i.e. if $h - l \geq (p_{hH} - p_{lH})\Delta_H + (p_{lL} - p_{hL})\Delta_L$.*

**Proof.** The proposition follows self-evidently from the derivation above. ■

## 6.4 Welfare Properties of the Separating Equilibrium

### 6.4.1 Model 1

The attractive property of the separating equilibrium is that in every period, all customers who buy the policy they are offered are made weakly better off than they would be under their outside option. Those who do not buy the policy are better off investing in their outside option.

In a pooling equilibrium, the number of customers who buy in the first period is either (weakly) greater or (weakly) less than the number who optimally ought to buy in the first period, depending on whether the true quality of the policy they are offered is $L$ or $H$. In the former case, some consumers who buy in the first period would have been better off investing in their outside option instead; in the latter case, some first-period non-buyers would have been better off buying the policy in the first period and furthermore, they cannot (under the rules of the game) buy in the second period either.

As discussed in section 6.1.3, the "renewal" of policies in the second period of our model is known in the insurance literature as persistency. The persistency rate is the ratio of investors who renew their policy. It is clear that in our two-quality model the persistency rate in a separating equilibrium is 100% for both high and low quality policies. In a pooling equilibrium (given that customers' expectation about policy performance is $0.5E[X|L] + 0.5E[X|H]$), the persistency rate for high quality policies is still 100%; for low quality policies, the persistency rate drops to $\frac{0.5E[X|H]-0.5E[X|L]}{0.5E[X|L]+0.5E[X|H]-p} < 1$.

Under the conditions identified in Corollary 54, the $H$-firm is also better off in a separating equilibrium. The $L$-firm, however, loses out: it will always be worse off in a separating than in a pooling equilibrium.

### 6.4.2 Model 3

Purely for the purpose of illustration, consider the simplified (martingale) version of model 3. In a separating equilibrium, if $x_0$ is $h$, period 0 demand is $q_0 = \frac{h}{p} - 1$, and expected period 1 demand is $q_1 = \frac{p_{hH}h+p_{hM}M+p_{hL}L}{p} - 1$. Therefore, $\frac{(1-p_{hH})h-p_{hM}M-p_{hL}L}{p}$ customers do not renew their policy at the end of period 0.

If $x_0$ is $l$, period 0 demand is $q_0 = \frac{l}{p} - 1$, and expected period 1 demand is $q_1 = \frac{(p_{lH}+p_{lM})l+p_{lL}L}{p} - 1$. Therefore, $\frac{p_{lL}l-p_{lL}L}{p}$ customers do not renew their policy at the end of period 0.

In a pooling equilibrium, assuming that buyers' priors on $x_0$ are $\Pr\{x_0 = h\} = \Pr\{x_0 = l\} = 0.5$, period 0 demand is $q_0 = \frac{0.5l+0.5h}{p} - 1$.

If $x_0$ is $h$, expected period 1 demand is $Eq_1 = \frac{p_{hH}(0.5h+0.5l)+p_{hM}\min\{0.5h+0.5l,M\}+p_{hL}L}{p} - 1$. Therefore, $\frac{0.5h+0.5l-(p_{hH}(0.5h+0.5l)+p_{hM}\min\{0.5h+0.5l,M\}+p_{hL}L}{p}$ customers do not renew their policy at the end of period 0.

If $x_0$ is $l$, expected period 1 demand is $Eq_1 = \frac{p_{lH}(0.5h+0.5l)+p_{lM}\min\{0.5h+0.5l,M\}+p_{lL}L}{p} - 1$. Therefore, $\frac{0.5h+0.5l-(p_{lH}(0.5h+0.5l)+p_{lM}\min\{0.5h+0.5l,M\}+p_{lL}L}{p}$ customers do not renew their policy at the end of period 0.

Comparison of non-renewals yields our final result:

**Corollary 57** *In the simple martingale example of model 3, persistency is higher in a separating equilibrium unless $x_0 = h$ and $M > 0.5h + 0.5l$.*

**Proof.** Compare the expected number of non-renewals in separating and pooling equilibrium.

For the case where $x_0 = l$, suppose first that $0.5h + 0.5l < M$. Then, a separating equilibrium exhibits higher persistency if $0.5(1 - p_{lH} - p_{lM})h + 0.5(1 - p_{lH} - p_{lM})l - (1 - p_{lH} - p_{lM})l \geq 0$, or $h - l \geq 0$, which is true by assumption. Suppose now that $0.5h+0.5l > M$. Then, a separating equilibrium exhibits higher persistency if $0.5(1 - p_{lH})h + 0.5(1 - p_{lH})l - p_{lM}M - (1 - p_{lH} - p_{lM})l \geq 0$. However, from $0.5h+0.5l > M$ (which we have just assumed), we know that

$$0.5(1-p_{lH})h+0.5(1-p_{lH})l-p_{lM}M-(1-p_{lH}-p_{lM})>$$

$$>0.5(1-p_{lH})h+0.5(1-p_{lH})l-(0.5h+0.5l)p_{lM}-(1-p_{lH}-p_{lM})l=$$

$$=0.5p_{lL}h-0.5p_{lL}l>0.$$

For the case where $x_0 = h$, suppose first that $0.5h + 0.5l > M$. Then, a separating equilibrium exhibits higher persistency if $0.5(l-h)+0.5p_{hH}(h-l) \geq 0$, or $1-p_{hH} \geq 0$, which is true by assumption. Suppose now that $0.5h + 0.5l < M$. Then, a separating equilibrium exhibits higher persistency if $0.5(1-p_{hH}-p_{hM})h+0.5(1-p_{hH}-p_{hM})l-h+p_{hH}h+p_{hM}M \geq 0$, or $0.5(h+l)p_{hL} - (p_{hL} + p_{hM})h + p_{hM}M \geq 0$. However, since $0.5(h+l) < M$, and $h > M$, in this case only, persistency is higher in a pooling than in a separating equilibrium. ∎

## 6.5 Conclusion

We began this discussion with the observation that, when a producer's marginal returns to advertising vary systematically with unobservable product quality, uninformative advertising (such as commission payments to retailers) may act as a signal of product quality. This paper has explored repeat purchases as the link between quality and returns to advertising, and found that, under the assumptions of proposition 53 and proposition 56, a separating equilibrium in the quality signalling game exists. Furthermore, these equilibria have desirable welfare properties.

In viewing advertising (or sales commission) as a quality signal, our paper has followed the lead of Milgrom and Roberts (1986). Interestingly, in a recent empirical study, Tomas, Shane, and Weigelt (1998) find that, in the US automobile market, prices and advertising expenditures are consistent with the "signalling" story.

In the present context, however, care must be taken not to overemphasize the importance of the signalling function of commission payments. In this paper, retailers serve no function other than that of conduit for commission payment signals of quality. However, as we have pointed out above, retailers may well acquire information about customers that producers want to extract. Commission payments may therefore be part of a mechanism designed by the producer in order to elicit this information. When the retailer is a common agent for more than one producer, the producers will typically compete in the mechanisms (the commission payment schedules) they design for the retailer. This, in essence, is the question addressed by the recent literature on common agency (cf. Martimort (1992), Stole (1992), and in particular, Mezzetti (1997)). For the reasons pointed out above, these concerns are largely orthogonal to the vertical differentiation issues of this paper. Nevertheless, making them explicit in the current context (with horizontal product differentiation) is a worthwhile future research project.

## 6.6 Appendix

This appendix contains figures referred to in section 6.1.3.

Figure 6.1: Market Share, New Yearly Premiums. Source: Association of British Insurers, October 1998, *Insurance: Facts, Figures and Trends*, London: ABI

Figure 6.2: Market Share, New Single Premiums. Source: Association of British Insurers, October 1998, *Insurance: Facts, Figures and Trends*, London: ABI

Figure 6.3: Empirical Frequency Distribution of Commission Payments: 25 Year Unit Linked Personal Pension. Source: Personal Investment Authority, January 1998, *Life Assurance Disclosure: Three Years On*, London: PIA

Figure 6.4: Empirical Frequency Distribution of Commission Payments: Unit Linked Whole of Life. Source: Personal Investment Authority, January 1998, *Life Assurance Disclosure: Three Years On*, London: PIA

Figure 6.5: Persistency rates of regular premium policies started in 1993. Source: Personal Investment Authority, January 1997, *Life Assurance Disclosure: Two Years On*, London: PIA

Figure 6.6: Persistency rates of regular premium policies started in 1993. Source: Personal Investment Authority, January 1997, *Life Assurance Disclosure: Two Years On*, London: PIA

# Chapter 7

# Who Regulates the Regulators?

In general, common agency models involve two distinct sources of inefficiency. One source of contractual inefficiency is one that is already present in the standard single-principal single-agent framework: As a result of the informational asymmetry between contract designer and decision-maker, the informed party (in this case, the agent) has to obtain an informational rent to supply effort (as in the moral hazard–hidden action model) or to reveal private information (as in the adverse selection–hidden action model).

The new source of contractual inefficiency introduced in common agency models arises as a consequence of the simultaneous, non-cooperative framework in which rival contract setters design mechanisms for a common agent. When the agent's actions (in the moral hazard model) or allocations (in the adverse selection model) are correlated,[1] the amount of informational rent given to the agent by one principal induces generally different (unwanted) behavior in the other principal's mechanism.

In this chapter we study this externality in a setting that differs from the standard common agency setting in one important respect. The model combines elements of both the pure common agency setting (Bernheim and Whinston (1986a), Martimort (1992), Stole (1992)) and a hierarchical principal-supervisor-agent model (e.g. Tirole (1986)). This greater richness allows us to address questions of common agency where one principal has greater power, or greater discretion, than the other. This richness however, comes at a price:

[1] Actions (allocations) in this context are correlated when a change in the action (allocation) under one principal's mechanism changes the agent's marginal cost of action (marginal utility of allocation) with respect to the action (allocation) under the other principal's mechanism.

we simplify the informational asymmetry issue by giving each principal the opportunity to buy a simple technology that reveals private information: higher precision is available at greater cost. This allows us to focus more clearly on the central issue of this chapter: the externality that arises from non-cooperative contracting.

## 7.1 Introduction

Since its inception in the 1970s, agency theory has provided new insights in virtually every branch of economics.[2] Arguably the most fruitful area of practical application has been to the economic analysis of regulation. As a result, we now have a good understanding of the incentive structures facing firms that are subject to regulatory scrutiny, as well as the incentives faced by their regulators. A large number of results are available, summarized in Laffont and Tirole's comprehensive text (Laffont and Tirole (1993); cf. also Laffont (1994)).

The economic analysis of competition policy, by comparison, is still in its infancy. For instance, it is much less clear in how far competition policy and its enforcement are subject to incentive pressures, or even what the appropriate action space for the competition authority is. In this paper, we focus interest on the regulatory function of the international competition authority; that is, we study the incentives that confront the international authority in its regulatory function. We will therefore refer to the international authority as an international regulatory agency. The problem is compounded if we consider semi-hierarchical structures, in which an international regulatory authority oversees not just the actions of the firms in an industry, but also those of the industry's national regulator. In this paper, we model competition law enforcement in such an hierarchical structure. Although ostensibly a model of a hierarchy, features of the common agency setting are preserved. In particular, both industry regulator and the international authority attempt to obtain, directly from the firm, information about the firm's privately known cost structure.

The paper studies the limits on the international authority's action space that arise from the externalities germane to common agency settings. The question we address in this paper is how much effort an authority should invest in obtaining information about the regulated firm (and, by implication, about the performance of the national regulator). The externality

[2] Chapters 3 and 4 are a guide to some of the results from this literature.

arises because, given sufficiently accurate information, the international regulatory authority always has an incentive to overrule the national regulator's decision and pursue its own agenda. As a result, increasing investigation effort by the international authority reduces the national regulator's incentive to investigate, and may lead to a "crowding out" of regulatory effort. In the context of the enforcement of competition law in the European Union, this effect is usually referred to, along with a host of others, under the heading of "co-operation" between member states and community authorities. In this paper, we isolate this effect, and introduce the "crowding out" terminology as a more informative term.

A case in point of the structure we study is the hierarchy of regulated firm–industry regulator–competition authority (European Commission) in the European Union, and our modelling choices reflect the institutional features of this setting. The following section therefore briefly reviews the structure of competition law enforcement in the EU.

### 7.1.1 Competition Law Enforcement in the EU

European antitrust law is based largely on Articles 81 and 82 of the EC Treaty (Treaty of Rome).[3,4] Article 81 (1) prohibits agreements between undertakings and "concerted practices" that affect trade between member states, and which "have as their object or effect the prevention, restriction or distortion of competition within the common market ..." (such as price-fixing agreements, input or output market-sharing agreements, price-discrimination agreements, etc.). Article 82 prohibits any abuse of a dominant position that affects trade between member states. Although Article 82 does not define what constitutes such abuse, it lists examples such as "unfair" (for instance, monopoly) pricing, price discrimination, restriction of production, or tying contracts. Furthermore, Article 86 of the EC Treaty brings state-owned firms, or those operating under a state-granted monopoly, under the umbrella of European antitrust law.

The application of European competition law is shared between the European competition authority (the European Commission, in this case Directorate General IV) and member states' national competition authorities. The competencies of national authorities

[3]Article 12 of the Treaty of Amsterdam, in force on May 1, 1999, provides for a renumbering of the Articles of the EC Treaty. In particular, Articles 85 and 86 are renumbered Articles 81 and 82. Article 90 is renumbered Article 86. We use the new (revised) numbering system.

[4]For an economic analysis of European competition policy, cf. Phlips (1995).

and the Commission are defined in Council Regulation 17, the first implementing regulation for Articles 85 and 86 (now Articles 81 and 82).[5] The Commission can act either on its own initiative (*ex officio*), or on application from interested parties or member states (Regulation 17 (3)). Importantly, as long as the Commission has not initiated a formal procedure that leads to the adoption of a decision, Regulation 17 (9) gives national competition authorities full competence to apply Articles 81 and 82.[6] However, provided that a case has European dimension (affects trade between member states), European Union law is in general understood to take precedence over national law.[7] This, of course, is in keeping with the principle of subsidiarity in Article 3 (b) of the EC Treaty. Finally, Regulation 17 (11) gives the Commission the right to obtain any necessary information from governments and competent authorities of the member states, as well as from firms directly. For this purpose, Regulation 17 (14) grants the Commission investigating powers and institutes a system of fines (Regulation 17 (15)) for non-compliance in investigation.

However, the Commission's powers to investigate are generally perceived to be severely limited by the fact that it has no powers against individuals;[8] it cannot, for instance, demand information from individuals, force individuals to testify, or impose fines on them. By contrast, all member states can request documents from and question individuals. Most member states can interrogate in written form, pursue on-site inspections and hold individual representatives of firms personally responsible for providing answers. In some member states (Finland, France, Germany, Greece, Ireland, the Netherlands, and the UK), non-compliance with the investigation may be punished by imprisonment with a maximum of between six weeks and two years; the European Commission may only impose fines which, particularly since they may only be imposed against undertakings, not individuals, are generally not considered to be high enough to deter non-compliance. This view is confirmed in the recent White Paper on the modernization of the implementation of European competition law:[9] it proposes amendment of Regulation 17 to allow for powers against persons in

[5] Council Regulation 17 of February 6, 1962 (OJ 13, 21.2.1962, p. 204; Special Edition 1959-62, p. 87)

[6] There is an exception: the granting of exemptions (individual and block exemptions) from the provisions in Article 85 (1) is a prerogative of the Commission.

[7] This principle was established in the European Court of Justice ruling in Case 14/68 Walt Wilhelm -v- Bundeskartellamt.

[8] cf. Laudati, L. (1995): "Surveys of Member States' Powers to Investigate and Sanction Violations of National Competition Law," *Competition Policy Newsletter* 1(4), 13–20.

[9] *White Paper on Modernisation of the Rules Implementing Articles 85 and 86 of the EC Treaty*, approved

investigations, and an increase in fines for supplying incorrect information (para. 113 and 124).

So there exists a clear hierarchy of competition law enforcement in the European Union: National authorities (which for the purposes of this paper we will take to be price-regulating industry regulators) have wide-ranging powers to investigate. If a violation of competition law is found, they can impose sanctions under Articles 81 and 82 or under national competition law where courts cannot directly implement European law. The European Commission has more limited powers of investigation, but may overrule the decisions of national authorities. The feature that national authorities have greater powers of investigation is interesting, since it opens up the possibility for an external effect: if the national authority investigates, this reveals information that is also at the Commission's disposal; if the national authority does not investigate, the Commission has less powerful tools to obtain information. In order to focus on this feature, we will model the structure of information about firm behavior as a pair of nested sets: In our model, the Commission's information is a subset of the information acquired by the national regulatory agency.

A closely related issue is that of co-operation between the Commission and national competition authorities. In its (1997) Notice,[10] the Commission defines the extent of co-operation in case allocation and regulatory decision-making between national authorities and the Commission, and again reinforces the principle of subsidiarity: in cases with a European dimension, there is a presumption that the Commission is competent. However, the Notice recognizes that duplication of investigation effort is possible, and, if it occurs, is costly and should "wherever possible be carried out by a single authority. ... Parallel proceedings ... can lead to the repetition of checks on the same activity, by the Commission, on the one hand, and by the competition authorities of the Member States concerned, on the other" (para. 10). However, the (1999) White Paper recognizes that Regulation 17 can not prevent simultaneous application of Articles 81 and 82 by national authorities and the European Commission in those countries that can implement European competition law

April 28, 1999.

[10] Commission Notice on Cooperation between National Competition Authorities and the Commission in Handling Cases Falling within the Scope of Articles 85 or 86 of the EC Treaty *OJ C 313*, October 15, 1997, p. 3.

directly.[11,12] Clearly, the lack of regulatory co-operation and co-ordination is perceived as a problem. In our model, we make this notion of the lack of co-operation precise: absence of co-ordinated competition law enforcement leads to what we call "crowding out" of effort by the national regulatory authority.

A final important issue is the availability of judicial review by (or, the availability of an avenue for appeal in) the Court of First Instance (Articles 172 and 173 of the Treaty of Rome and Regulation 17 (17)). The regulated firm has the option to challenge any Commission action, decision, or the imposition of penalties. The Court of First Instance has full competence to change any penalties or fines imposed by the Commission. In the context of our model, this condition ensures that regulatory and competition authorities will only take actions based on "hard" information (that is, information that is verifiable in court). In other words, we rule out the possibility that either the national regulator, or the international competition authority can regulate the firm on the basis of a guess (however correct) about the firm's cost structure. Any such decision could successfully be challenged in court by the regulated firm.

Importantly for our purpose, Article 82 allows the European Commission to act as a regulatory authority. The Commission has used Article 82 to regulate price directly in two cases: In a 1974 decision[13] the Commission used Article 82 to impose a fine on General Motors Continental (GMC) for its practice of charging excessive prices for motor vehicle inspections. Belgian law requires all motor vehicles to be inspected and issued with a certificate of conformity. The Commission took the view that GMC charged excessive prices for the inspection of parallel imports. Although, on appeal, the fine was annulled by the court,[14] the court's decision was based on the fact that the Commission had not sufficiently shown an abuse of prices. In fact, the court stated explicitly that an abuse of

[11] Currently only 8 member states' competition authorities can apply European competition law directly (Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal, and Spain). The remaining member states apply their own antitrust law.

[12] Cf. also the view of the Director-General, Alexander Schaub, in (1998) "EC Competition System—Proposals for Reform," *Fordham Corporate Law Institute: 25th Annual Conference: International Antitrust Law & Policy*: "... a number of obstacles to an efficient decentralisation and closer co-operation still remain. ... national and EC competition law can be applied in parallel by national competition authorities and the Commission. ... [A] clear division of work ... presupposes that all Member States empower their competition authorities to apply EC competition law."

[13] Commission Decision of December 19, 1974 (OJ L29, 3.2.1975, p. 14)

[14] Judgement of the Court of November 13, 1975 (Case 26/75)

a dominant position in the sense of Article 82 "might lie, *inter alia*, in the imposition of a price which is excessive in relation to the economic value of the service provided."

In a 1975 decision[15] the Commission again sought to impose a penalty for excessive pricing. In this case, United Brands Company (UBC) had charged widely diverging wholesale prices for its imports of bananas into different member states. Again, the Commission's decision was rejected on appeal, but again not because it was inadmissible for the Commission to regulate prices, but because the Commission had failed to make out a clear case. In fact, the court decided that "charging a price which is excessive because it has no reasonable relation to the economic value of the product supplied may be an abuse of a dominant position within the meaning of subparagraph (a) of Article [82]" and points clearly to markup over cost as a measure for this abuse: "this excess could, *inter alia*, be determined objectively if it were possible for it to be calculated by making comparison between the selling price of the product in question and its cost of production, which would disclose the amount of the profit margin."[16]

We take this evidence to support our view that price regulation is a valid field of intervention for the European Commission. We will therefore sometimes refer to the international competition authority as an international regulatory authority.

### 7.1.2 Modelling Strategy

We model the regulated firm as a monopolist with a privately known cost parameter. The firm, which sells both in home and foreign markets is assumed to be a profit-maximizing single-price monopolist. This assumption is for convenience only: price discrimination issues needlessly complicate the analysis.

Both the national regulator and the international regulatory authority can obtain information about the firm's cost parameter by investing in a stochastic information-revelation technology. We model the fact that national regulators have "better" information than the international regulatory authority (because of the regulators' greater power of investigation) in the following way: while regulators have powers to investigate firms directly, the international authority may only request information from national regulators. The information

[15] Commission Decision of December 17, 1975 (OJ L95, 9.4.1976, p. 1)
[16] Judgement of the Court of February 14, 1978 (Case 27/76)

potentially available to the international authority is therefore a subset of the regulator's information.

In general, the objectives of regulator and international authority will not coincide; furthermore, both the regulator's and the international authority's objective functions will differ from the firm's. This divergence of interests drives the common agency aspect of the problem: both authorities wish to obtain information about the firm's privately known cost parameter, and both wish to regulate the firm in different ways. In this paper, we make strong simplifying assumptions about the regulator's and the competition authority's objective function, but these can be viewed as reduced form results from, for instance, a model of interest group influence. In particular, we assume that the firm is interested in profit maximization; the international competition authority, in maximizing total (consumer and producer) surplus; and the regulator in maximizing a weighted sum of producer and consumer surplus.[17] In the light of a theory of interest group influence we could justify these choices as arising from the fact that regulators have a concern for a future career in the regulated industry (that is, their objective is partly aligned with that of the firm), but need to signal their ability to the regulated firm by displaying competence in applying competition rules (so that part of their objective is welfare maximizing). Alternatively, the regulator may have an interest in a future career in government and therefore aligns its objective with that of the government. Since governments tax firms, but are elected by agents interested in their consumer surplus, we can again justify the regulator's objective as a weighted sum of consumer and producer surplus. There is now a growing literature on the "capture" of the regulatory agency by the regulated firm or other interest group. Much of this literature has assumed enforceable side contracts with direct money payments such as campaign contributions (for instance, Laffont and Tirole (1991)), but recently interest has shifted to other motivating factors such as career concerns. This is already implicit, for

[17] In the case of the European Commission it is not clear what the objective is. Here we focus on efficiency (total surplus) as the objective. One might equally well argue for consumer protection (consumer surplus) as the objective. In fact, the objectives underlying European competition law are often inconsistent: although consumer protection is an important objective, the protection of "small and medium-sized undertakings" or the creation of a single market (for instance by allowing exclusive distributors in order to allow firms to penetrate a new market) are also important aims (cf. Whish (1993)). At any rate, all we need for our results is that the objective of the national industry regulator is between that of the international authority and that of the firm, so that there is some coincidence of interests between the international authority and the national regulator.

instance, in the papers by Spiller (1990), and Che (1995) (reviewed below). The starting point for the latter branch of the literature are the papers by Dewatripont, Jewitt and Tirole (Dewatripont, Jewitt, and Tirole (1999a), Dewatripont, Jewitt, and Tirole (1999b)). We make no assumption about which of these factors motivate regulators or international authorities, but find it helpful to keep in mind a career concern model. At any rate, the aim of this paper is to study the externalities that arise from non-cooperative contracting, not the modelling of interest group pressure.

The firm sets price to maximize profits. Given sufficiently accurate information, regulator and international authority can intervene and set the price that maximizes their objective. Given the hierarchical structure of competition law enforcement, if an authority that ranks higher in the hierarchy intervenes, the price it sets will be binding. To justify this assumption, we appeal to the principle of subsidiarity in European competition law enforcement, and in particular, to the European Court of Justice ruling (Walt Wilhelm) mentioned above, which establishes legislative priority of European courts over legislative authorities in member states.

Since price is the choice variable for regulator and international authority, one may wonder whether the regulated firm may have an incentive to decrease output below the level of demand at the set price. This case can be ruled out through an appeal to a universal service obligation embedded in the licence of virtually every privatized utility, and which requires that every customer wishing to be served at the set price not be refused service.

Furthermore, as indicated above, regulator and international authority can only act on "hard" information about the firm's cost parameter. They cannot, for instance, set price based on an expected value of firm cost. This feature of the model reflects the possibility of judicial review that was argued above: the action of each agency has to be able to stand up in court.

### 7.1.3 Results

The externality we study in this paper operates in two directions: as the regulator acquires more information, the opportunity for the international authority to obtain hard information is raised. However, the more the international authority investigates, the greater the probability that the regulator's effort results in price-setting less prefered by the regulator, and therefore the regulator's incentive to acquire information is reduced.

One of the results of the paper therefore is a counterintuitive conclusion: by introducing an international authority to "regulate the regulators," consumers may get a worse deal than if industry regulators operated independently. The trade-off is the following: When the international authority investigates successfully, consumers get the best possible deal (marginal cost pricing). But investigation by the international authority (through the crowding out mechanism) reduces the probability that the firm's behavior is investigated successfully by anyone and therefore increases the probability that the firm's price is set. This of course is the worst possible outcome for consumers. As a result, the expected price of the firm's product may be higher than if the regulator were to operate independently. More pertinently, expected consumer surplus may also be lower than if the regulator were to work independently.

The paper's central prediction is that we can derive bounds on the regulatory authority and competition authority behavior we should expect to observe. In particular, a robust prediction is that the international authority will never fully crowd out regulatory effort.[18] More subtly, in a wide class of cases the international authority will never investigate only tentatively: it will either not investigate at all, or invest substantial effort into its investigation of the firm's cost structure. We make this statement more precise by studying a special case of our more general model. Further, an important variable in the model is the size of the country relative to the size of the area of responsibility of the international regulatory authority. This allows us to make country-specific predictions of national regulatory and international authority behavior. In particular, in small countries, neither the regulator nor the international authority will investigate fully. In large countries, regulatory effort will always be maximal, regardless of the actions of the international authority.

### 7.1.4 Related Literature

Our model combines elements from common agency and hierarchical "supervision" models. The common agency aspect enters through the competition for information about the

[18] A caveat is necessary: we show that regulatory effort will never be zero, unless it would be zero even in the absence of an international authority. In this sense, the international authority never causes complete crowding out.

firm's cost (which allows setting of the price preferred by each principal) between regulator and international authority. Both principals have different objectives, and each principal's objective differs from that of the regulated firm.

Spiller (1990) has a common agency model (in a hidden action setting) of competition between congress and an organized interest group ("industry") for favorable pricing decisions by the regulator. The paper is reviewed in section 3.4 of chapter 3. In Spiller's paper, congress and industry bid (through budget-setting and direct money transfers, respectively) for favorable price outcomes. The informational asymmetry is about unobservable regulator actions which induce a distribution over observable pricing outcomes. Spiller's paper differs from ours both in the informational structure of the problem it studies, and in emphasis: while Spiller explores the objective of the regulator, and how regulatory actions are influenced by political pressure, we emphasize the problems antitrust enforcement agencies (in their price regulating role) with differing incentives create for each other in extracting information about a firm subject to regulation.[19]

In this sense, our model is closer to that of Martimort (1996b) (cf. section 4.7 of chapter 4). Martimort models two regulatory agencies that non-cooperatively choose subsidies for a firm with privately known cost of performing a project beneficial to the constituencies of both regulators. The firm chooses whether or not to invest in the project. Martimort shows that under non-cooperative contracting, the project is less likely to be performed than under co-operative or full information assumptions on the model. In Martimort's model, as in ours, regulators impose externalities on each other: one regulator's subsidy has external benefits (if the project is performed) for the other regulator's constituency. Both regulators, however, are identical: there is no sense of one regulator having greater discretion or power.

In many applications, there is no such symmetry of regulatory power; often one authority is subordinated (if sometimes only partially) to another. At the same time, these principals may maximize their own objectives, which depend on the agent's private information in opposite ways. This is the case for the regulatory example we address in this paper, but

[19] Spiller's paper is interesting because his empirical discussion finds career concerns (post-regulatory employment opportunities in regulated firms) as an important motivational factor for regulators. His theoretical model, however, does not reflect this: the modelling is still in terms of direct money payments (cf. our brief discussion of the career concerns literature of Dewatripont, Jewitt, and Tirole (1999a) and Dewatripont, Jewitt, and Tirole (1999b), above).

the structure is that of a whole class of problems. (For instance, while college students and university administration lobby instructors for opposite amounts of effort in teaching, it is the university that has greater "power" to determine actual teaching output.) We find this aspect of reality interesting and therefore choose to introduce elements of hierarchical decision-making into our model.

Tirole (1986) develops an hierarchical "principal-supervisor-agent" model, where the agent produces (observable) output with privately known effort, according to some stochastic technology. The principal obtains the proceeds from selling the agent's output and pays the agent a wage contingent on output and a report on the agent's performance from a supervisor. While the agent knows the size of the productivity shock in her production technology, the supervisor has coarser information. Specifically, the supervisor receives a signal that either reveals the productivity variable or else reveals nothing. The supervisor makes a report to the principal, in which she can either report the state of the productivity variable truthfully or report that she has observed nothing, but cannot "lie" directly. This is the extent of the supervisor's discretion, and it drives the model's results. The supervisor is rewarded by the principal on the basis of her report and the productive output of the agent.

The focus of Tirole's paper is to describe the optimal collusion-proof contract, that is the mechanism that prevents agent and supervisor from exchanging side transfers. In general, the possibility of side contracts (although, in equilibrium, no side contracts are actually made), implies that the supervisor will use her discretion to act as an advocate for the agent, i.e. she will sometimes hide information detrimental to the agent: the principal would be better off if side contracts could be prevented costlessly. Yet, there is a role for the supervisor: if the agent could produce verifiable information herself, she would only ever choose to reveal information that is not detrimental to her. Tirole argues that organizational design is partly a response to the threat of collusion; organizations are designed so as to minimize the possibility of side transfers. The possibility of collusion (and the relative ease of preventing it in short-run versus long-run relationships) serves as an explanation for why we observe short-run relationships, rather than long-run relationships that generally allow more investment in relationship-specific assets. Furthermore, bureaucratic rules leave no discretion to the enforcer (they take away the supervisor's discretion to hide information unfavorable to the agent), so that the possibility of collusion may explain the emergence of bureaucracies.[20]

In our paper, we use part of this hierarchical structure. In particular, we make use of the nested information structure between agent and supervisor. In our own model, the regulator can acquire (unlike in Tirole's paper, at a cost) an imperfect signal of the firm's cost parameter, and it can control the accuracy of the signal. We then duplicate this structure again and allow the competition authority to acquire at a cost an imperfect signal of the regulator's signal. Unlike in Tirole's paper, where the supervisor determines what the principal knows directly, we allow the competition authority to determine what information it obtains—but the opportunities for this information acquisition are restricted by the regulator's choice of how much (and how precise) information it acquires.

An example of a hierarchical model in the field of regulation is the paper by Che (1995), which applies Tirole's framework to the study of regulatory capture by the firm. In a hierarchy of regulated firm (agent), regulator (supervisor), and government (principal), Che studies the question of whether the existence of a "revolving door" (post-government employment opportunities in the regulated industry for the regulator, and the opportunity for collusion between regulator and regulated firm) always needs to be harmful. In fact, Che finds that the revolving door can give the regulator the (*ex ante*) incentive to acquire industry-specific human capital, and to regulate the industry strictly (*ex post*) in order to signal its qualification for post-regulatory employment in the industry.

A further application, and extension to a three-tier hierarchy, of Tirole's model is the paper by Kofman and Lawarrée (1993), which models an organization designer's choice between an internal (cheap but collusion-prone) auditor and a combination of internal and external (truthful but costly) auditors. Note that in our model the analogue to Kofman and Lawarrée's auditor is the regulatory authority. In our model the regulator is, by assumption as by law (Regulation 17 (11)), truthful.[21]

[20]Cf. also the more extensive discussion in Tirole (1992).

[21]How accurate is the assumption of truthfulness? One might envision a regulator who conceals bad (or biased) performance by misreporting information on cost. In this paper, we abstract from this possibility because our interest is in the contractual externality between principals. Chapter 5 presents a model where an investigator might want to obtain less information on cost in order to be able to overreport credibly about a firm's cost.

### 7.1.5 Outline

The next section describes the model. Section 7.3 contains the main results of the paper. We solve the general model for optimal national regulatory and international authority effort and present results about these optimal choices. In particular, we derive our main proposition placing a bound on possible international authority behavior. Finally, we engage in a comparative statics exercise to derive results about outcomes for countries of different sizes. Section 7.4 sharpens some of our results for special cases of the more general model. We refine the bound derived in section 7.3 to show that, under certain conditions on regulator and international authority cost-of-effort functions, regulatory effort will always exceed $\frac{1}{2}$. That is, regulators will intervene in price setting in at least half of all cases that they investigate, and in which the firm prices excessively. Section 7.5 discusses extensions to the model and explains some of our modelling choices in greater depth. The final section 7.6 concludes.

## 7.2 The Model

The regulated firm is a monopolist of privately known unit cost $\theta$. Since we are not interested in market structure issues we assume, without loss of generality, the absence of fixed costs. The monopolist produces both for home consumption and for export, and we assume that price discrimination is impossible. The firm faces a downward-sloping, differentiable, aggregate demand curve $q(p)$, and in entirely conventional fashion is assumed to maximize profit $\pi(p,\theta)$. Assume further that the fraction of output sold in the home market is proportional to the size of the home market, relative to the size of the export markets.[22] Define $f(p,\theta,\alpha,\beta) = \alpha\pi(p,\theta) + \beta\int_p^\infty q(t)dt$. The monopolist therefore seeks to maximize $f(p,\theta,1,0)$.

The firm is regulated by a regulator whose objective is a weighted sum of the firm's profits and consumer surplus. One way to justify this assumption is to suppose that the regulator is motivated by a concern for a future career in government. Government cares both about the firm's profit (because it can be taxed) and about the fraction of consumer surplus that is enjoyed by its own nationals. On this interpretation, $\alpha < 1$ reflects the

[22] Given a representative country $i$ of size $\beta_i$ ($\sum\beta_i = 1$), this assumption says that $\beta_i q(p) = q_i(p)$.

shadow cost of taxation, and $\beta$ denotes the relative size of the country (the extent to which consumer surplus figures in the objective function). The regulator seeks to maximize $f(p, \theta, \alpha, \beta) = \alpha\pi(p, \theta) + \beta \int_p^\infty q(t)dt$. This interpretation also requires that $\alpha > \beta$.[23] We model the regulator as regulating price directly: we ignore any incentive issues that in practice drive a wedge between direct price regulation, rate-of-return regulation and price-cap regulation. Furthermore, price is the only policy variable for the regulator.[24] A universal service obligation, for instance, ensures that the firm has to produce on its demand curve, that is, it cannot refuse customers who are willing to buy the firm's product at the set price.

The regulator can invest (costly) effort $e \in [0, 1]$ in an investigation technology that yields a signal $\sigma \in \{\theta, \emptyset\}$ of the firm's cost parameter. The regulator learns the firm's cost parameter with probability $\Pr\{\sigma = \theta | e\} = e$, and with probability $\Pr\{\sigma = \emptyset | e\} = 1 - e$ it learns nothing. The technology is available at cost $c_r(e)$, with standard assumptions on the cost function: $c_r(0) = 0$, $c_r'(\cdot) > 0$ at all but a finite number of points (for instance, we will allow $c_r'(0) = 0$), $c_r''(\cdot) \geq 0$, and these derivatives exist everywhere.[25] The interpretation of effort as the probability of "success" (obtaining verifiable information about the firm's cost parameter) will be useful in the interpretation of our results.

The international authority's objective, because it represents the interests of consumers in all countries under its jurisdiction, is the maximization of total surplus; that is, if it intervenes it sets price so as to maximize $f(p, \theta, 1, 1)$. Similarly to the regulator, it has the option to invest effort $i \in [0, 1]$ in a costly technology that reveals a signal $s \in \{\theta, \emptyset\}$ such that $\Pr\{s = \sigma | i\} = i$, and with probability $\Pr\{s = \emptyset | i\} = 1 - i$. This is the previously argued assumption of the nestedness of information: the international authority can only learn firm cost if the regulator has received an informative signal. The information that the authority can buy is coarser than the regulator's: and its coarseness varies with the regulator's investigation effort. Effort level $i$ costs the international authority $c_c(i)$ (again assuming $c_c(0) = 0$, $c_c'(\cdot) > 0$ at all but a finite number of points, $c_c''(\cdot) \geq 0$, and that these

[23] In fact, this is also a technical assumption: it implies that if the regulator sets price, it will not choose a price at which the firm finds it unprofitable to produce any positive level of output.

[24] Issues such as quality monitoring and quality assurance open up a can of worms of their own. In this paper, we focus on price regulation and therefore ignore such issues, important as they are in practice.

[25] Throughout, subscript $r$ denotes parameters or variables in the regulator's utility function. Subscript $f$ denotes the firm's and subscript $c$ the international regulatory (competition) authority's parameters or variables.

derivatives exist everywhere).

Note that none of our results require the specific form for the payoff function $f(p, \theta, \alpha, \beta)$ that we have assumed. Any separable function of the form $\alpha g(p, \theta) + \beta h(p, \theta)$ (this includes $g(p, \theta)^{\alpha} h(p, \theta)^{\beta}$) that have a positive maximizer will do equally well. The restrictions that are necessary are that $\alpha g(p, \theta) + \beta h(p, \theta)$ be concave for all values of $\beta$ (so that a maximum exists) and, for lemma 63, that $\frac{\partial h(p,\theta)}{\partial p} < 0$. This lends considerable generality to our model.

### 7.2.1 Timing

The timing of the game between firm, industry regulator, and international authority is the following:

1. The firm learns its unit production cost $\theta$.

2. The international authority decides bindingly (and makes publicly known) its investigation probability, $i$. For instance, it commits to a certain staff size, budget, or sets public targets.

3. The regulator decides its regulatory effort, $e$, and learns signal $\sigma$ of firm cost.

4. The international authority learns its signal $s$, with the characteristics discussed above.

5. The firm privately decides its output price, $p_f(\theta)$.[26]

6. If the regulator has received an informative signal ($\sigma = \theta$), it privately decides its preferred price for the firm's output, $p_r(\theta)$. Otherwise, it does nothing.

7. If the international authority has received an informative signal ($s = \theta$), it privately decides its preferred price for the firm's output, $p_c(\theta)$. Otherwise, it does nothing.

8. If neither regulator nor international authority have intervened, price $p_f$ is published. If only the regulator has intervened, price $p_r$ is published. If both regulator and international authority have intervened, price $p_c$ is published. Firm, regulator, and international authority receive their payoffs.

[26] For expositional purposes, we will occasionally suppress the dependence of prices on firm cost in notation.

## 7.3 Solving the Model

We solve for the subgame perfect equilibrium of this game by backward induction.

First, we find the Nash (actually, the dominant strategy) equilibrium in the price-setting game. When setting price, each agent's dominant strategy is to set price to maximize its objective.[27] Any other price, if it is published as the final price, gives the price-setter a lower payoff. But price-setting does not influence the probability of seeing one's preferred price published as the final price, so each agent's dominant strategy has to be to set the price that maximizes its payoff. Thus, the firm sets $p_f(\theta) = \arg\max_p f(p, \theta, 1, 0)$, the regulator sets $p_r(\theta) = \arg\max_p f(p, \theta, \alpha, \beta)$, and the international authority sets $p_c(\theta) = \arg\max_p f(p, \theta, 1, 1)$.

### 7.3.1 The Regulator's Effort Choice

The regulator chooses effort $e$ as a function of the international authority's choice of $i$. It chooses its effort to:[28]

$$\max_e E_\theta \left[(1-e) f(p_f, \theta, \alpha, \beta) + e\left(i f(p_c, \theta, \alpha, \beta) + (1-i) f(p_r, \theta, \alpha, \beta)\right)\right] - c_r(e).$$

(That is, with probability $(1-e)$ it will obtain no information about the firm's cost, and therefore the firm's price prevails. With probability $e$, either of two things happen: with probability $i$, the international authority also obtains hard information and consequently sets $p_c$, or with probability $(1-i)$, the international authority obtains no hard information about firm cost and therefore the regulator's price prevails.)

[27] This is a feature specific to this model. For instance, if the timing were changed so that the regulator sets price before the international authority chooses effort and this price were observable, the regulator's price would likely act as a signal about the nature of its information. Similarly, the unobservability of the firm's price is important: since the firm's demand curve is known, cost could be calculated from the optimal price choice. This assumption is less controversial than it looks: the possibility of judicial review requires that national regulatory or international authority decisions are based on verifiable information about cost. In practice, the relationship between observed price, demand and underlying cost is made opaque by issues such as nonlinear pricing, allocation of cost to different products in a multiproduct firm, etc.

[28] Since $e$ is bounded between 0 and 1, the regulator in fact chooses $e(i) = \arg\max_e \max\{\min\{1, E_\theta[...] - c_r(e)\}, 0\}$. Because of the concavity of $E_\theta[...] - c_r(e)$, we can first maximize with respect to $e$ and then take account of the boundaries later. This simplifies the exposition considerably.

The first-order condition defines the function $\hat{e}(i)$ such that

$$c_r'(\hat{e}(i)) \equiv E_\theta \left[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta) - i\left(f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right)\right]. \quad (7.1)$$

First note that, absent an international authority, the regulator would set effort such that

$$c_r'(\hat{e}) \equiv E_\theta \left[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)\right].$$

Comparison of this first order condition and that in equation (7.1) reveals the nature of the externality the international authority imposes on the regulator: since $f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta) \geq 0$ ($p_r$ is the maximizer of $f(p,\theta,\alpha,\beta)$), the existence of the externality from the international authority generally tends to reduce regulatory effort.[29] Bearing in mind the interpretation of $e$ as a probability of successful investigation, this illustrates the hypothesis that, if there is an international authority, consumers may, on occasion, get a worse pricing deal than without an international authority.

The first-order condition (7.1) gives us our first proposition about the shape of the regulator's optimal effort response function in the presence of an international authority:

**Proposition 58** *For $p_r \neq p_c$, the function $\hat{e}(i)$ is strictly decreasing if $c_r''(\cdot) > 0$. If $p_r = p_c$, $\hat{e}(i)$ is constant if $c_r''(\cdot) > 0$.*

**Proof.** Equation (7.1) holds as an identity and can therefore be differentiated. Differentiation with respect to $i$ yields

$$\hat{e}'(i)c_r''(\hat{e}(i)) = -E_\theta \left[\left(f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right)\right].$$

Since $p_r$ maximizes $f(p_r,\theta,\alpha,\beta)$, we have $E_\theta \left[\left(f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right)\right] \geq 0$. If $p_r \neq p_c$, $E_\theta \left[\left(f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right)\right] > 0$, so that $\hat{e}'(i) < 0$ if $c_r''(\cdot) > 0$. The result for $p_r = p_c$ follows immediately. ■

Proposition 58 is a "crowding out" proposition: It says that, the higher the international authority's investigation effort is, the less effort the regulator will invest in obtaining

[29] Note, however, that $\hat{e}$ is bounded between 0 and 1, so that this externality need not always reduce regulatory effort in fact.

information about firm behavior. This proposition is intuitive: The higher the international authority's effort, the greater the probability that it will set its own price. The regulator can reduce this probability by reducing its own effort.

Note that the function $\hat{e}(i)$ is not compatible with the interpretation of $e$ (or $i$) as probabilities: both points in its domain and in its range may lie outside the unit interval. The concavity of the regulator's "unconstrained" objective however simplifies our task: we define a function $e(i)$ that takes on value 1 when $\hat{e}(i) > 1$, value $\hat{e}(i)$ when $0 \leq \hat{e}(i) \leq 1$, and value 0 when $\hat{e}(i) < 0$. By concavity of the regulator's objective, this is the best "constrained" response. Since the function $e(i)$ is bounded between 0 and 1, and $\hat{e}(i)$ is monotone decreasing everywhere, there are at most three regions on function $e(i)$, as follows:

1. $i \in \left[0, \max\left\{0, \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}\right\}\right)$ characterized by $e(i) \equiv 1$;
2. $i \in \left[\max\left\{0, \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}\right\}, \min\left\{\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(0)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}, 1\right\}\right]$ characterized by $e'(i) < 0$, with $e(i) = \hat{e}(i)$ as in equation (7.1);
3. $i \in \left(\min\left\{\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(0)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}, 1\right\}, 1\right]$ characterized by $e(i) \equiv 0$.

Note that this function $e(i)$ is continuous (although not everywhere differentiable), and weakly monotone decreasing.

In region 1, optimal regulatory effort is high enough for increased international authority activity not to discourage effort. In region 2, there is some crowding out of regulatory effort (as established in Proposition 58). Region 3 is a region of "total crowding out:" international authority activity is so great that regulatory effort is fully discouraged.

### 7.3.2 The International Authority's Effort Choice

The international authority's objective is to:

$$\max_i E_\theta[i\,(e(i)f\,(p_c,\theta,1,1) + (1-e(i))\,f\,(p_f,\theta,1,1)) +$$
$$+ (1-i)\,(e(i)f\,(p_r,\theta,1,1) + (1-e(i))\,f\,(p_f,\theta,1,1))] -$$
$$-c_c(i).$$

(That is, with probability $(1 - e(i))$ the regulator obtains no information, and the firm's price prevails. With probability $ie(i)$ both regulator and international authority obtain hard information, in which case the international authority sets its price. With probability $(1 - i)e(i)$ the regulator, but not the international authority, obtains verifiable information about firm cost, and $p_r$ is set.)

Let $u(i)$ denote this objective function. We now study this objective separately for all three regions, and investigate the international authority's optimal effort choice.

**Region 1:**

For $i \in \left[0, \frac{E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)] - c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)]}\right)$, we have $e(i) \equiv 1$. The international authority's objective in this region is therefore to

$$\max_i E_\theta \left[ i f(p_c, \theta, 1, 1) + (1 - i) f(p_r, \theta, 1, 1) \right] - c_c(i),$$

with the first-order condition

$$c_c'(i^*) = E_\theta \left[ f(p_c, \theta, 1, 1) - f(p_r, \theta, 1, 1) \right].$$

This allows us to state our next result, which gives conditions for the international authority's objective to be decreasing in region 1 (if region 1 exists):

**Proposition 59** *For sufficiently large countries ($\beta$ sufficiently large), or for cost-of-effort functions that are sufficiently steep at the origin ($c_c'(0)$ sufficiently large), the international authority will not investigate at all.*

First, we need the following lemma (recall the restriction that $\beta < \alpha$):

**Lemma 60** *As $\beta \to \alpha$, $p_r(\theta) \to p_c(\theta)$.*

**Proof.**

$$\begin{aligned}
\lim_{\beta\to\alpha} p_r(\theta) &= \\
&= \lim_{\beta\to\alpha} \arg\max_p \alpha\pi(p,\theta) + \beta \int_p^\infty q(t)dt \\
&= \lim_{\beta\to\alpha} \arg\max_p \pi(p,\theta) + \frac{\beta}{\alpha} \int_p^\infty q(t)dt \\
&= \arg\max_p \pi(p,\theta) + \int_p^\infty q(t)dt \\
&= p_c(\theta),
\end{aligned}$$

This proves the lemma. ■

The lemma is intuitive: it states the simple fact that if a country is large (in the limit, the country is the only country in the union), the regulator's objective coincides with the international authority's objective. In this limiting case, their pricing behavior is therefore identical.

We are now in a position to prove proposition 59.

**Proof.** The international authority's objective is monotone decreasing over the entirety of region 1, so that the optimal choice of $i$ in this region is at the boundary $i^* = 0$, if

$$c_c'(0) \geq E_\theta \left[f\left(p_c,\theta,1,1\right) - f\left(p_r,\theta,1,1\right)\right].$$

Since $p_c$ is the maximizer of $f(p,\theta,1,1)$, we have $E_\theta \left[f\left(p_c,\theta,1,1\right) - f\left(p_r,\theta,1,1\right)\right] \geq 0$. By lemma 60, we have $\lim_{\beta\to\alpha} E_\theta \left[f\left(p_c,\theta,1,1\right) - f\left(p_r,\theta,1,1\right)\right] = 0$. In region 1, therefore, the international authority will wish to set $i^* = 0$.

Next, we need to prove that if $c_c'(0) \geq E_\theta \left[f\left(p_c,\theta,1,1\right) - f\left(p_r,\theta,1,1\right)\right]$, the international authority's objective in region 2 is also monotone decreasing.

In region 2, the international authority's objective has slope

$$\begin{aligned}
&E_\theta[(e(i) + ie'(i))(f(p_c,\theta,1,1) - f(p_r,\theta,1,1)) + \\
&+e'(i)(f(p_r,\theta,1,1) - f(p_f,\theta,1,1))] - c_c'(i).
\end{aligned}$$

From $c_c'(0) \geq E_\theta \left[f\left(p_c,\theta,1,1\right) - f\left(p_r,\theta,1,1\right)\right]$ and $c_c''(\cdot) > 0$, we know that $c_c'(i) \geq$

$E_\theta[f(p_c,\theta,1,1)-f(p_r,\theta,1,1)]$. But this implies that the international authority's objective has a slope less than

$$E_\theta[(e(i)-1+ie'(i))(f(p_c,\theta,1,1)-f(p_r,\theta,1,1))+$$
$$+e'(i)(f(p_r,\theta,1,1)-f(p_f,\theta,1,1))].$$

Since we know that $E_\theta[f(p_c,\theta,1,1)-f(p_r,\theta,1,1)] \geq 0$ and $E_\theta[f(p_r,\theta,1,1)-f(p_f,\theta,1,1)] \geq 0$, and $e(i) \leq 1$, by $e'(i) < 0$ (in region 2), it follows immediately that the international authority's objective in region 2 is monotone decreasing. ■

Under the conditions in proposition 59, the international authority will never wish to investigate tentatively: it will either set $i^* = 0$ (i.e. not investigate at all), or set $i^*$ somewhere in region 2 (i.e. investigate "rigorously").[30]

We can also work out sufficient conditions under which the international authority will choose to set $i$ at the right region 1 boundary $i^* \to \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$. We refer to this as investigating "rigorously."[31]

**Proposition 61** *If region 1 exists, the international authority will always investigate "rigorously" if it has*

(a) *a linear cost-of-effort function from the family characterized by* $c_c'(i) < E_\theta[f(p_c,\theta,1,1)$ $-f(p_r,\theta,1,1)]$ *for all* $i$ *in region 1;*

(b) *a quadratic cost-of-effort function from the family characterized by* $c_c'(i) = ai^b$ *for all* $i$ *in region 1, with* $b > 1$, $0 < a < \frac{1}{b}\frac{E_\theta[f(p_c,\theta,1,1)-f(p_r,\theta,1,1)]}{\left(\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}\right)^{b-1}}$;

(c) *more generally a cost function with the following restriction on its concavity:* $\int_0^1 c''(t)dt \leq E_\theta[f(p_c,\theta,1,1)-f(p_r,\theta,1,1)]$.

**Proof.** Part (a) of the proposition follows immediately. For part (b) note that continuity and concavity of the authority's objective guarantee that, if $c_c'(0) < E_\theta[f(p_c,\theta,1,1)-$

[30] We show below as a very general proposition that the competition authority will never choose to set $i$ in (or even near) region 3.

[31] This nomenclature may be misleading: if region 1 does not exist, the proposition notwithstanding, the competition authority may not investigate at all if the objective in region 2 is decreasing.

$f(p_r, \theta, 1, 1)]$ and $c_c'\left(\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}\right) \leq E_\theta[f(p_c, \theta, 1, 1) - f(p_r, \theta, 1, 1)]$, the objective will be monotone increasing over the entirety of region 1. All we require from a quadratic function is therefore that

$$ab\left(\frac{E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)] - c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)]}\right)^{b-1} \leq E_\theta[f(p_c,\theta,1,1) - f(p_r,\theta,1,1)].$$

Since, by assumption $\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]} \geq 0$, the expression in the proposition follows. Part (c) of the proposition immediately follows from the requirement that $c_c'(1) \leq E_\theta[f(p_c,\theta,1,1) - f(p_r,\theta,1,1)]$. ■

**Region 2:**

For $i \in \left[\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}, \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(0)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}\right]$, $e(i)$ is defined through

$$c_r'(e(i)) \equiv E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta) - i(f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta))].$$

In this region, the international authority's objective has slope

$$\begin{aligned}&E_\theta[(ie'(i) + e(i))(f(p_c,\theta,1,1) - f(p_r,\theta,1,1)) + \\ &+e'(i)(f(p_r,\theta,1,1) - f(p_f,\theta,1,1))] - \\ &-c_c'(i).\end{aligned}$$

This gives us the following result:

**Proposition 62** *Within region 2, if it exists, the international authority will never set* $i^* = \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(0)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$, *i.e. it will never locate at the right boundary of region 2.*

The proposition establishes that in region 2, where $e(i)$ is negative monotone, the international authority is always better off allowing at least some regulatory effort: it will locate to the left of the point where $e(i) = 0$. But first we need a lemma.

**Lemma 63** $E_\theta[f(p_r,\theta,1,1) - f(p_f,\theta,1,1)] \geq 0.$

**Proof.** The proof proceeds in two steps. Note first that $p_f$, $p_r$ and $p_c$ can be ordered, as follows: $p_f \geq p_r \geq p_c$. This follows from the maximization problem that $p_f$, $p_r$ and $p_c$ solve:

$$\begin{aligned} p_f &= \arg\max_p \pi(p,\theta) \\ p_r &= \arg\max_p \pi(p,\theta) + \frac{\beta}{\alpha}\int_p^\infty q(t)dt \\ p_c &= \arg\max_p \pi(p,\theta) + \int_p^\infty q(t)dt. \end{aligned}$$

Since $\int_p^\infty q(t)dt$ is a decreasing function of $p$, the ranking of $p_f$, $p_r$ and $p_c$ follows.

Next, we need to show that this ranking implies a ranking over $f(p,\theta,1,1)$. But this is straightforward: since $f(p,\theta,1,1)$ is concave in $p$, and is maximized by $p_c$, the ranking of $p_f$, $p_r$ and $p_c$ implies that $f(p_r,\theta,1,1) \geq f(p_f,\theta,1,1)$. ■

Now we can prove proposition 62:

**Proof.** At $i = \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(0)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$, $e(i) = 0$. Since $e'(i) < 0$ in region 2, and since $E_\theta[f(p_c,\theta,1,1) - f(p_r,\theta,1,1)] \geq 0$ and (by lemma 63) $E_\theta[f(p_r,\theta,1,1) - f(p_f,\theta,1,1)] \geq 0$, and since $c_c'(\cdot) > 0$, we know that at the right boundary of region 2, we have $\frac{du}{di}\left(\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(0)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}\right) < 0$, so that moving $\varepsilon$ to the left increases the international authority's objective. ■

This proposition, again, is intuitive: moving away from the point where regulatory effort is fully crowded out gives the international authority an increased probability of winning out over the regulator in its price-setting. Furthermore, such a move to the left reduces costly effort, so it must be worthwhile.

**Region 3:**

For $i \in \left(\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(0)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}, 1\right]$, we have $e(i) \equiv 0$. The international authority's objective in this region is therefore to

$$\max_i E_\theta [f(p_f,\theta,1,1)] - c_c(i).$$

This of course has a boundary solution, at $i^* \to \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(0)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$. The reason for this is entirely intuitive: where regulatory effort is zero, the information collected by the regulator is never informative. By implication, and as a result of the informational structure of the model, the international authority never receives hard information about firm cost. In this case, reducing international authority effort does not decrease the information available to the international authority (and therefore does not lead to a worse outcome for the authority), but it reduces effort cost. This argument establishes that it is never optimal for the international authority to locate on the inside of region 3.

Together with the previous result about region 2, this gives us immediately the following simple, and very general:

**Proposition 64** *It is never in the international authority's interest to discourage all regulatory effort.*

**Proof.** The proposition follows immediately from the discussion in the text, and from proposition 62. ■

### 7.3.3 Size Effects

The previous section has asked questions about the optimal international authority effort. The maintained hypothesis has been that all three regions (region 1 of no crowding out, region 2 of partial crowding out, and region 3 of full crowding out) exist. The robust conclusion was that the international authority will never crowd out regulatory effort fully. Furthermore, in a large number of cases, international authority behavior is discontinuous: either the authority will not investigate at all, or it investigates rigorously. We will study this last proposition in the context of a special case more rigorously below.

First, however, we need to understand the comparative statics of the boundaries between regions. In this section, we ask how country size interacts with our predictions. We focus on determining the relative size of regions 1, 2, and 3 along the $e(i)$ function, and study how these regions vary with country size.

We have the following limit result about the size of region 1:

**Proposition 65** *For sufficiently (not only vanishingly) small countries, region 1 does not exist (so that the international authority's effort always crowds out some regulatory effort). For large countries, if region 1 ever exists, it becomes large (so that no regulatory effort is crowded out).*

First, we need the following lemma that states that regulators in very small countries set prices like firms:

**Lemma 66** *As $\beta \to 0$, $p_r(\theta) \to p_f(\theta)$.*

**Proof.**

$$\begin{aligned}
\lim_{\beta\to 0} p_r(\theta) &= \\
&= \lim_{\beta\to 0} \arg\max_p \alpha\pi(p,\theta) + \beta \int_p^\infty q(t)dt \\
&= \lim_{\beta\to 0} \arg\max_p \pi(p,\theta) + \frac{\beta}{\alpha} \int_p^\infty q(t)dt \\
&= \arg\max_p \pi(p,\theta) \\
&= p_f(\theta).
\end{aligned}$$

This proves the lemma. ■

We are now in a position to prove proposition 65:

**Proof.** The boundary between region 1 and region 2 is $i = \frac{E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)] - c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)]}$. Since, by the previous lemma (lemma 66), $\lim_{\beta\to 0} p_r(\theta) = p_f(\theta)$, for small countries this boundary converges to $\lim_{\beta\to 0} \frac{E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)] - c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)]} = -\frac{c_r'(1)}{E_\theta[f(p_f,\theta,\alpha,0) - f(p_c,\theta,\alpha,0)]}$. But since $f(p_f,\theta,\alpha,0) = \alpha f(p_f,\theta,1,0)$, and $p_f$ is the maximizer of $f(p_f,\theta,1,0)$, we know that $E_\theta[f(p_f,\theta,\alpha,0) - f(p_c,\theta,\alpha,0)] \geq 0$. Furthermore, by $c_r'(0) \geq 0$ and $c''(\cdot) > 0$ we know that $c_r'(1) > 0$. Therefore, $-\frac{c_r'(1)}{E_\theta[f(p_f,\theta,\alpha,0) - f(p_c,\theta,\alpha,0)]}$ is a negative number for all cost functions. By continuity of $\frac{E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)] - c_r'(1)}{E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)]}$ in $\beta$ (for $\beta < \alpha$), the first part of the proposition follows for sufficiently (not only vanishingly) small countries.

For the second part of the proposition, note that we know from lemma 60 that for large countries, $p_r(\theta) \to p_c(\theta)$, and therefore $E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)] \to 0$. Therefore, if region 1 ever exists (i.e. $E_\theta[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)] - c_r'(1) > 0$), region 1 grows without bound for sufficiently large $\beta$. ■

Now that we know that region 1 becomes small for sufficiently small countries, we may ask what fills the void. The following proposition is again intuitive: for very small countries, regulatory and firm pricing behavior is identical, so that the regulator (and therefore, by implication, the international authority) will withhold all effort:

**Proposition 67** *For small countries, region 3 becomes large (neither regulator nor international authority will intervene).*

**Proof.** Recall that the left region 3 boundary is at $i = \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(0)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$. We know from lemma 66 that, as country size becomes small, $p_r(\theta) \to p_f(\theta)$, so that $\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-c_r'(0)}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$ becomes a nonpositive number (since $c_r'(0) \geq 0$, and $E_\theta [f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)] \geq 0$). Therefore, region 3 becomes large. We know what this implies about regulatory behavior since, by definition, in region 3 $e(i) = 0$. The implication for international authority behavior follows from the obvious fact that if $e(i) = 0$, reducing $i$ increases the international authority's objective. ∎

It is worth noting that this last result applies for sufficiently (not only vanishingly) small countries if $c_r'(0)$ is sufficiently large.

### 7.3.4 Price Distribution

The regulator's and the international authority's effort choices induce a probability distribution over prices. Without an international authority, price $p_r$ will realize with probability $e(0)$, and price $p_f$ with probability $1 - e(0)$. With international authority, the distribution is $\{(p_c, i^*e(i^*)), (p_r, (1-i^*)e(i^*)), (p_f, (1-e(i^*))\}$. Coming back to the question about whether consumers necessarily get a better deal when there is an international authority, we can now compare expected prices. In particular, price on average will be higher with an international regulatory authority when

$$e(0) > e(i^*)\left(1 + i^* \frac{p_c - p_r}{p_r - p_f}\right). \tag{7.2}$$

Clearly, if the international authority's effort does not crowd out any regulatory effort ($e(0) = e(i) = 1$), average price when there is an international authority is lower than without international authority. Equally obviously, at the point of total crowding out ($e(i) = 0$), the average price with international authority (which is then just $p_f$) is higher than without (when it is a, possibly degenerate, mixture between $p_f$ and $p_r$). When $e(i)$ is decreasing (in region 2), it is straightforward that the function on the right-hand side of inequality (7.2) tends to first increase with increasing $i$ and then decrease. Whether the latter effect is sufficiently strong (and whether $i^*$ can be sufficiently large) is an empirical question.

More pertinent, the effort choices of national regulatory and international authorities induce a distribution over consumer surplus. Making use of our notation, consumer surplus from price $p$ is $f(p, \theta, 0, 1)$. Without an international authority, surplus $f(p_r, \theta, 0, 1)$ will realize with probability $e(0)$, and surplus $f(p_f, \theta, 0, 1)$ with probability $1 - e(0)$. With an international regulator, the distribution is $\{(f(p_c, \theta, 0, 1), i^* e(i^*)), (f(p_r, \theta, 0, 1), (1 - i^*)e(i^*)), (f(p_f, \theta, 0, 1), (1 - e(i^*))\}$. Average consumer surplus will be higher with an international regulatory authority than without when

$$\begin{aligned} & e(0)f(p_r, \theta, 0, 1) + (1 - e(0))f(p_f, \theta, 0, 1) \\ > \; & i^* e(i^*)f(p_c, \theta, 0, 1) + (1 - i^*)e(i^*)f(p_r, \theta, 0, 1) + (1 - e(i^*)f(p_f, \theta, 0, 1). \end{aligned} \tag{7.3}$$

At $i^* = 0$, this inequality obviously holds as an equality. The right-hand side of inequality (7.3) is a function of $i^*$. As $i^*$ increases, this function has slope

$$\begin{aligned} & i^* e'(i^*)(f(p_c, \theta, 0, 1) - f(p_r, \theta, 0, 1)) - e'(i^*)(f(p_f, \theta, 0, 1) - f(p_r, \theta, 0, 1)) + \\ & + e(i^*)(f(p_c, \theta, 0, 1) - f(p_r, \theta, 0, 1)). \end{aligned}$$

Since $f(p, \theta, 0, 1)$ is decreasing in $p$, we know that the first two terms in this sum are nonpositive, and the last is nonnegative.

In region 1, where $e(\cdot) = 1$ (so that $e'(\cdot) = 0$), this slope is therefore $f(p_c, \theta, 0, 1) - f(p_r, \theta, 0, 1) \geq 0$. We know from proposition 65 that, for large countries, if region 1 ever exists, it becomes large. We also know that, for large countries, $p_r \rightarrow p_c$, so that $f(p_c, \theta, 0, 1) - f(p_r, \theta, 0, 1) \rightarrow 0$. Therefore, if region 1 ever exists, for large countries, expected consumer surplus is the same both with and without an international authority.

If region 1 does not exist, then we have the result that, at least for sufficiently large countries (where $p_r \rightarrow p_c$), this slope is negative, so that inequality (7.3) holds. In this illustrative case, expected consumer surplus is lower when an international authority "regulates the regulator."

### 7.3.5 Discussion

The nature of the externality between regulator and international authority lies in the efficiency of the information flow from regulator to international authority. In our model, greater international regulatory authority effort leads to regulatory "crowding out" because it reduces the probability that the regulator sees its preferred price imposed on the firm. At the same time, regulatory effort determines the quality of information available to the international authority. We have chosen to model this informational assumption such that the information potentially available to the international regulatory authority is a subset of the national regulator's information. The feature of differential quality of information is crucial to our model: as argued above, it opens up the possibility for potentially damaging external effects. Modelling the quality of information as two nested sets is clearly an extreme modelling choice. However, it is obvious that the main features of our model rely only on the relative coarseness of the international regulatory authority's information and will therefore survive a less extreme modelling assumption.

## 7.4 Some Special Cases

We can obtain sharper predictions about national regulatory and international authority behavior by placing restrictions on the cost functions of national regulator and international regulatory authority. First consider a regulator with constant unit cost of effort.

### 7.4.1 Case 1

Consider a regulator with constant unit cost of effort, $c_r(e) = ke$. This gives us an affine objective function which has a boundary solution. This regulator will therefore choose $e = 1$ if

$$E_\theta \left[ if\left(p_c, \theta, \alpha, \beta\right) + (1-i)\, f\left(p_r, \theta, \alpha, \beta\right)\right] - k \geq E_\theta \left[f\left(p_f, \theta, \alpha, \beta\right)\right],$$

and $e = 0$ otherwise. This gives us the following step function $e(i)$:

$$e(i) = \begin{cases} 1 \text{ for } i \leq \frac{E_\theta\left[f(p_r,\theta,\alpha,\beta) - f\left(p_f,\theta,\alpha,\beta\right)\right] - k}{E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right]} \\ 0 \text{ for } i > \frac{E_\theta\left[f(p_r,\theta,\alpha,\beta) - f\left(p_f,\theta,\alpha,\beta\right)\right] - k}{E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right]} \end{cases}.$$

Note also that this result is a limit result for the general $e(i)$ function derived in section 7.3: for $c_r(e) = ke$, $c_r'(0) = c_r'(1) = k$, so that the boundaries between region 1 and region 2 and between region 2 and region 3 coincide. The linear cost function cuts out region 2.

Note also that in this case, the international authority's objective will be discontinuous at the point $\hat{\imath} = \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-k}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$:

$$\begin{aligned} &\lim_{i\to\hat{\imath}^-} E_\theta[i\,(e(i)f\,(p_c,\theta,1,1) + (1-e(i))\,f\,(p_f,\theta,1,1)) + \\ &+ (1-i)\,(e(i)f\,(p_r,\theta,1,1) + (1-e(i))\,f\,(p_f,\theta,1,1))] - c_c(i) \\ = \;& E_\theta[\hat{\imath}f\,(p_c,\theta,1,1) + (1-\hat{\imath})\,f\,(p_r,\theta,1,1)] - c_c(\hat{\imath}) \end{aligned}$$

and

$$\begin{aligned} &\lim_{i\to\hat{\imath}^-} E_\theta[i\,(e(i)f\,(p_c,\theta,1,1) + (1-e(i))\,f\,(p_f,\theta,1,1)) + \\ &+ (1-i)\,(e(i)f\,(p_r,\theta,1,1) + (1-e(i))\,f\,(p_f,\theta,1,1))] - c_c(i) \\ = \;& E_\theta[\hat{\imath}f\,(p_f,\theta,1,1) + (1-\hat{\imath})\,f\,(p_r,\theta,1,1)] - c_c(\hat{\imath}) \end{aligned}$$

Since $p_c$ maximizes $f(p,\theta,1,1)$, we know that $f(p_c,\theta,1,1) \geq f(p_f,\theta,1,1)$, so that the "step" at $\hat{\imath} = \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-k}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$ is a "step down." Further, since the international authority's objective is monotone decreasing for $i > \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-k}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$ (by the familiar argument: regulatory effort is zero, so increasing international authority effort brings no benefits, but is costly), we know that the objective for $i > \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-k}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$ can never be greater than for $i \leq \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-k}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$. Therefore, the behavior of the objective in region 1 determines entirely where (in region 1) the international authority will locate. It will never set $i^* > \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-k}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$.

If the international authority's cost function is also linear, $c_c(i) = \kappa i$, we can say more about the international authority's behavior. We already know that we need only study the objective function in region 1, where $e(i) \equiv 1$. With linear cost, we will obtain a boundary solution, so that the international authority will choose $i^* = 0$ if $\kappa > E_\theta[f\,(p_c,\theta,1,1) - f\,(p_r,\theta,1,1)]$ and will choose $i^* = \frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]-k}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$ otherwise.[32]

[32]This last conclusion is robust: Whenever the international authority has a linear cost function, its behavior in region 1 will be at the boundary: either it will withhold all effort, or it will investigate with

This case gives us a polarized result: Either the international regulatory authority never intervenes, or it intervenes with a probability such that the regulator is just indifferent between investing full and investing no effort. That is, we should observe the international authority either to do nothing, or to "throw the book" at a regulated firm. Further, in equilibrium we would always expect to see the regulator dedicate full effort to investigation of the firm's cost structure. Recalling the interpretation of effort as the probability of "success" (obtaining verifiable information), the testable implication of this result is that we should always see the national regulator (or the international authority) trumping the firm's own price.

The regulator's cost-of-effort function of course depends on the industry it regulates. For instance, a firm producing a large number of differentiated goods (so that cost allocation is an issue) is "harder" to regulate than a single-good monopolist; a cost-structure highly sensitive to random factors is "harder" to investigate than one that is not; etc. The case of linear cost-of-investigation, while clearly extreme, may be a pointer to one of the reasons why we would always expect certain industries to be regulated, while others are only subject to regulator-imposed pricing from time to time.

### 7.4.2 Case 2

We know that the international authority will never want to locate in region 3. If it locates in region 2, can we say more about where precisely? Given a set of assumptions on the national regulatory and international regulatory authority cost-of-effort functions, we obtain a sharp result.

Consider the case where both regulator and international authority have a quadratic cost-of-effort function, $c_r(x) = c_c(x) = \frac{1}{2}x^2$. About this case we have the following surprising result:[33]

**Proposition 68** *With quadratic cost-of-effort, the upper bound on international authority effort is half the right boundary of region 2.*

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effort level of at least the level at the right boundary of region 1.

[33]One may ask whether the restriction of the cost-of-effort function to $c_r(e) = \frac{1}{2}e^2$, and $c_c(i) = \frac{1}{2}i^2$ is essential, rather than choosing $ke^2$ ($\kappa i^2$, respectively). The answer is no. As can easily be ascertained, as long as $k > 0$, $\kappa > 0$, the result still holds. $c_r(e) = \frac{1}{2}e^2$, and $c_c(i) = \frac{1}{2}i^2$ are chosen for presentational ease.

**Proof.** With quadratic cost-of-effort, the right boundary of region 2 is $\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$. From the regulator's maximization problem, given quadratic cost, we obtain immediately

$$\hat{e}(i) \equiv E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)\right] - iE_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right],$$

with

$$\hat{e}'(i) \equiv -E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right].$$

Recall the international authority's objective $u(i)$, with derivative:

$$\begin{aligned}&\frac{du(i)}{di} = E_\theta[\left(ie'(i) + e(i)\right)\left(f(p_c,\theta,1,1) - f(p_r,\theta,1,1)\right) + \\ &+e'(i)\left(f(p_r,\theta,1,1) - f(p_f,\theta,1,1)\right)] - \\ &-c_c'(i).\end{aligned}$$

Substituting into this, we have as a first-order condition for maximization of $u(i)$ in region 2:

$$\begin{aligned}0 &= \{E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)\right] - 2i^* E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right]\} \cdot \\ &\quad E_\theta[f(p_c,\theta,1,1) - f(p_r,\theta,1,1)] - \\ &\quad -E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right] E_\theta[f(p_r,\theta,1,1) - f(p_f,\theta,1,1)] - i^*,\end{aligned}$$

or

$$\begin{aligned}i^* &= \frac{E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)\right] E_\theta[f(p_c,\theta,1,1) - f(p_r,\theta,1,1)]}{1 + 2E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right] E_\theta[f(p_c,\theta,1,1) - f(p_r,\theta,1,1)]} - \\ &\quad - \frac{E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right] E_\theta[f(p_r,\theta,1,1) - f(p_f,\theta,1,1)]}{1 + 2E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_c,\theta,\alpha,\beta)\right] E_\theta[f(p_c,\theta,1,1) - f(p_r,\theta,1,1)]}.\end{aligned}$$

Consider the two summands separately. Since $E_\theta\left[f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)\right] > 0$ and $E_\theta[f(p_c,\theta,1,1) - f(p_r,\theta,1,1)] > 0$ (by the now familiar argument from maximization) we have $\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]E_\theta[f(p_c,\theta,1,1)-f(p_r,\theta,1,1)]}{1+2E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]E_\theta[f(p_c,\theta,1,1)-f(p_r,\theta,1,1)]} < \frac{1}{2}\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$.[34] For essentially the same reason, and making use of lemma 63, we also know that $\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]E_\theta[f(p_r,\theta,1,1)-f(p_f,\theta,1,1)]}{1+2E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]E_\theta[f(p_c,\theta,1,1)-f(p_r,\theta,1,1)]} > 0$. The conclusion, therefore, that $i^* < \frac{1}{2}\frac{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_f,\theta,\alpha,\beta)]}{E_\theta[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)]}$ follows directly.

That this is indeed the maximum, not a minimum, can be seen from the second-order condition, which holds:

$$\begin{aligned}\frac{d^2u(i)}{di^2} &= E_\theta[(ie''(i)+2e'(i))\,(f(p_c,\theta,1,1)-f(p_r,\theta,1,1))+\\ &\quad +e''(i)\,(f(p_r,\theta,1,1)-f(p_f,\theta,1,1))]-c_c''(i)\\ &= E_\theta[2e'(i)\,(f(p_c,\theta,1,1)-f(p_r,\theta,1,1))-c_c''(i)<0.\end{aligned}$$

(The equality follows because, with quadratic cost-of-effort, $e(i)$ is linear, so that $e''(i)=0$; the inequality follows since in region 2 $e'(i)<0$, $f(p_c,\theta,1,1)-f(p_r,\theta,1,1)>0$, and $c_c''(i)>0$.) ■

This result is surprising: it allows us to make the sharp prediction that the international authority will never wish to reduce the regulator's effort below $e=\frac{1}{2}$.[35] This follows from the fact that, with quadratic cost-of-effort, the $\hat{e}(i)$ function is affine; since it decreases linearly from a value of one to a value of zero over the length of region 2, and we know that the international authority will never wish to locate more than halfway into region 2 (depending on the size of region 1 possibly much less), it will never crowd out regulatory effort to a point below $e=\frac{1}{2}$.

Again, the testable implication is that we should expect the regulator (or the international authority) to "win out" over the firm in at least (and likely more than) half the number of cases in which the regulator starts an investigation.

## 7.5 Further Issues

This section discusses some extensions of the model. First, we simulate a simple numerical version of special cases 1 and 2. We then consider the case in which the international

[34]The expression is of the form $\frac{ac}{1+2bc}=\frac{1}{\frac{1}{ac}+2\frac{b}{a}}<\frac{1}{2\frac{b}{a}}=\frac{1}{2}\frac{a}{b}$. The inequality follows for $ac>0$.

[35]Unless, because of country size effects, the regulator's effort in the absence of an international authority would be below $e=\frac{1}{2}$.

authority sets its investigation effort for more than one country, but cannot discriminate in its allocation of effort between countries. The following subsection remarks briefly on opportunities for collusion in our model. Finally, we discuss in more detail the informational structure in our model, and why we have chosen to model informational opportunities as a pair of nested sets.

### 7.5.1 Numerical Examples

Some of the results from section 7.4 can be simulated. We use a linear demand curve, $q(p) = a - bp$, and a uniform prior distribution over cost ($\theta$ is $U[0, \frac{a}{b}]$).[36] We can illustrate the special cases (case 1 and case 2, above) for specific, purely illustrative, parameter values. All figures are in the appendix to this chapter.

**Case 1:**

Figure 7.1 illustrates, for a linear cost-of-effort function for the regulator (case 1) of $c_r(e) = ke$, and for parameter values $a = 20$, $b = 1$, $\alpha = 1$, $\beta = 0.5$, $k = 1$, the graph of $e(i)$, and, for a linear cost-of-effort function for the international authority of $c_c(i) = \kappa i$, and values of $\kappa$ ranging between 1 and 9, the graphs of $u(i)$. Note the predicted discontinuity in $u(i)$ and the predicted polarization of international authority effort (i.e. either $i^* = 0$ or $i^* = \frac{E_\theta\left[f(p_r,\theta,\alpha,\beta)-f\left(p_f,\theta,\alpha,\beta\right)\right]-k}{E_\theta\left[f(p_r,\theta,\alpha,\beta)-f(p_c,\theta,\alpha,\beta)\right]}$).

**Case 2:**

Figure 7.2 presents, for parameter values $a = 20$, $b = 1$, $\alpha = 1$, and for $\beta$ ranging between 0.25 and 0.95, the regulator's optimal response function $e(i)$, and the international authority's objective. Kinks indicate the boundaries between regions 1 and 2 and regions 2 and 3. Figure 7.2 illustrates the proposition that the international authority never wishes to locate in region 3, that regulatory effort will never be below $\frac{1}{2}$, and that comparative statics on country size (varying $\beta$) have the predicted effect on the relative sizes of regions 1 and 3. Experimentation with different values of $a$, $b$, and $\alpha$ revealed no qualitatively different results.

[36] We choose this distribution for convenience: the assumption implies that the highest cost firm can choose the monopoly markup over cost and yet produce nonnegative output.

### 7.5.2 No Country-by-Country Discrimination

In our discussion in sections 7.3 and 7.4 we assumed that the international authority could set its optimal investigation strategy tailored to each country it was investigating. In practice, it seems unlikely that international authorities can discriminate so finely in the allocation of effort across countries.

From what we know from section 7.3, for small countries, the size of region 3 is relatively large; for large countries, region 1 becomes large (if it ever exists). We also know (from proposition 64) that the international authority never wishes to crowd out all regulatory effort if it can tailor its investigation effort to each country; that is, it will investigate relatively little in the small country (in which region 3, the region of full crowding out, is large). We have also found that, within region one (which is large for large countries), the authority will investigate either not at all or with some degree of rigor. For illustration, consider the case in which, in the large country, the authority investigates "rigorously." In this case it is straightforward that, if the international authority is restricted to setting the same level of investigation effort $i$ for both countries, it will investigate relatively less in the large country, and investigate relatively more in the small country. The problem here might be that for the small country, regulatory effort is already low (if no region 1 exists), so that international authority investigation at a higher level may crowd out national regulatory effort completely.

### 7.5.3 Collusion

Hierarchical principal-supervisor-agent models allow the study of collusion, and the effect this possibility has on the equilibrium allocation of effort and monetary transfers (cf. the discussion in section 7.1.4, above). Since our model incorporates elements from the literature on hierarchies, the question of the possibility of collusion in our context naturally arises.

In our model, we have excluded the possibility of collusion. We have ignored this possibility in the present paper because the enforceability of collusive side-contracts in the setting that we chose to model (that of European competition law enforcement) is at least questionable. A few remarks on collusion, however, are in order.

Clearly, there is scope for collusion in our model: the firm, for instance, could pay the regulator not to acquire information, and in exchange agree to lower its price. The question then is: how does the regulator know that the firm is keeping its part of the (side) contract? The conjecture is that some regulatory information-gathering will still be optimal, and in this case, the externality between national regulator and international regulatory authority that we focus on in this paper continues to arise. The other option for the firm would be to bribe the regulator not to reveal its information to the international authority. We assume that this is effectively prevented by the setup of our problem: the European Commission can, by Regulation 17 (11), require national antitrust enforcement agencies to make available all relevant information.

Is there an incentive for international authority and regulator to collude (i.e. to co-operate)? For instance, the international authority may want to pay the regulator to obtain (and share) full information, and then agree to regulate at a price between its own preferred price and the regulator's preferred price. The collusively set price would then maximize the sum of regulator's and international authority's payoff, that is it would solve

$$p_{coll}(\theta) = \max_{p} f(p, \theta, \alpha, \beta) + f(p, \theta, 1, 1) - c_r(1) - c_c(0).$$

(Note that by assumption $c_c(0) = 0$.) The scope for collusion then depends on whether there exists some side transfer $t$ that makes both the regulator better off than it would be in equilibrium, i.e.

$$\begin{aligned} & E_\theta \left[ f(p_{coll}, \theta, \alpha, \beta) \right] - c_r(1) + t \\ \geq \ & E_\theta \left[ (1 - e(i^*)) f(p_f, \theta, \alpha, \beta) + e(i^*) \left( i^* f(p_c, \theta, \alpha, \beta) + (1 - i^*) f(p_r, \theta, \alpha, \beta) \right) \right] - \\ & - c_r(e(i^*)), \end{aligned}$$

and that makes the international authority better off than in equilibrium:

$$\begin{aligned} & E_\theta \left[ f(p_{coll}, \theta, 1, 1) \right] - t \\ \geq \ & E_\theta [ i^* \left( e(i^*) f(p_c, \theta, 1, 1) + (1 - e(i^*)) f(p_f, \theta, 1, 1) \right) + \\ & + (1 - i^*) \left( e(i^*) f(p_r, \theta, 1, 1) + (1 - e(i^*)) f(p_f, \theta, 1, 1) \right) ] - c_c(i^*). \end{aligned}$$

This defines an interval within which such a transfer payment has to reside.

One factor that makes the existence of such a transfer more likely is the case where, *ceteris paribus*, the regulator's equilibrium effort is low. This is the case, for instance, in small countries (proposition 65). In this case, the international authority has much to gain from bribing the regulator to acquire, and share, full information. A cynical interpretation of this result is that it suggests that small member states of the European Union tend to receive higher economic aid subsidies in part because their national regulatory authorities would not otherwise spend sufficient investigation effort.

Similarly, an obvious result is that, in the limit, as regulator's and international authority's preferences are perfectly aligned (so that we have $\alpha = \beta = 1$; $p_r = p_c = p_{coll}$; $e(\cdot) = 1$, from proposition 65; and $i^* = 0$), there will of course always exist such a $t$. In the limit, we obtain

$$0 \leq t \leq c_c(i^*).$$

Continuity ensures that some $t$ like this also exists for sufficiently large countries.

### 7.5.4 Information Structure

We have chosen a very specific structure of the interaction of the timing of the model and the availability of information to regulator and international authority. In particular, we have assumed that the international regulatory authority can only observe the firm's cost when the regulator has received "hard" information about firm cost. This created the nested information structure that this paper has exploited. Clearly, other modelling choices could have been made.

A limiting case is that of informational independence: Whether or not the regulator observes firm cost (it observes cost with probability $e$ and does not observe cost with probability $(1-e)$), the international authority observes cost with probability $i$. The regulator therefore seeks to

$$\begin{aligned}
& \max_e E_\theta[(1-e)\left(if\left(p_c,\theta,\alpha,\beta\right) + (1-i)\,f\left(p_f,\theta,\alpha,\beta\right)\right) + \\
& +e\left(if\left(p_c,\theta,\alpha,\beta\right) + (1-i)\,f\left(p_r,\theta,\alpha,\beta\right)\right)] - c_r(e) \\
= \; & \max_e E_\theta\left[if(p_c,\theta,\alpha,\beta) + (1-e)(1-i)\,f\left(p_f,\theta,\alpha,\beta\right) + e\,(1-i)\,f\left(p_r,\theta,\alpha,\beta\right)\right] - c_r(e),
\end{aligned}$$

with the first-order condition

$$c_r'(\hat{e}(i)) = E_\theta \left[ (1-i) \left( f(p_r, \theta, \alpha, \beta) - f(p_f, \theta, \alpha, \beta) \right) \right].$$

Again, we obtain an externality from international authority to regulator: higher international authority effort implies lower regulatory effort. By comparison with equation (7.1), we see that whether regulatory effort is crowded out more strongly in this model depends on the relative sizes of $E_\theta [f(p_r, \theta, \alpha, \beta) - f(p_f, \theta, \alpha, \beta)]$ and $E_\theta[(1-i)(f(p_r, \theta, \alpha, \beta) - f(p_c, \theta, \alpha, \beta))]$. But there is no externality on the international authority comparable to that examined in this paper: the international authority's information set depends only on its own effort choice, not on that by the regulator (although the externality through the pricing implication of the regulator's lower effort persists). In general, in a model like this one should expect greater international authority involvement. One would use this modelling approach if both the international authority and the regulatory authorities had identical powers of investigation. As we have seen in section 7.1.1, in the European Union this is not the case. It is for this reason that our interest in this paper is in modelling externalities imposed by two competing principals on each other, and hence we have chosen the nested information structure above.

More generally, the information structure under which the international authority operates is, of course, a choice variable for the designer of the hierarchical structure (the legislator; for instance, parliament). Independent investigation by the international regulatory authority is, of course, costly: it loses information already acquired by national authorities. This opens up the larger question of how hierarchies should be structured optimally. While this paper has explored some of the implications of one specific structure (nested information), this larger question is beyond the scope of this paper.

A further modelling option would have been to give the international regulatory authority a certain amount of autonomy in its investigation: While the regulator's effort influences the probability of success of the international authority's effort, the international authority may be able to obtain hard information, even if the regulator has happened not to observe firm cost. We could model this as follows: The signal $s \in \{\theta, \emptyset\}$ that the international authority obtains could be such that $\Pr\{s = \emptyset | e, i\} = e \cdot i$, so that the probability distribution over pricing outcomes is $\{(p_c, e\cdot i), (p_r, e(1-e\cdot i)), (p_f, (1-e)(1-e\cdot i))\}$. The regulator therefore seeks to

$$\max_e E_\theta\left[(1-e)(1-ei)f(p_f,\theta,\alpha,\beta) + e(1-ei)f(p_r,\theta,\alpha,\beta) + eif(p_c,\theta,\alpha,\beta)\right] - c_r(e),$$

with the first-order condition

$$c_r'(\hat{e}(i)) = (2\hat{e}(i)i - i - 1)f(p_f,\theta,\alpha,\beta) + (1-2\hat{e}(i)i)f(p_r,\theta,\alpha,\beta) + if(p_c,\theta,\alpha,\beta).$$

The resulting function $\hat{e}(i)$ has derivative

$$\frac{d\hat{e}(i)}{di} = \frac{(f(p_c,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)) - 2\hat{e}(i)(f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta))}{c_r''(\hat{e}(i)) + 2i(f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta))}$$

Here we cannot even prove in general that regulatory effort is crowded out—this depends on the size of $2\hat{e}(i)(f(p_r,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta))$ relative to $f(p_c,\theta,\alpha,\beta) - f(p_f,\theta,\alpha,\beta)$.[37] This also illustrates the increase in the model's computational complexity at, presumably, little informational benefit.

We have chosen to focus on an admittedly limiting case of nested information sets because we believe it brings out an important aspect of the structure of European competition law enforcement: the relative poverty of the Commission's means of investigation, relative to those possessed by member states. In our model, all the international authority can do is request material previously acquired by antitrust enforcement agencies in member states. We believe that our results are still informative of actual decision mechanisms.

### 7.5.5 Future Research

In this paper, we have studied the hierarchy of a national industry regulator and an international authority that engages in price regulation. An interesting extension of this problem is to model the hierarchy of a national and an international antitrust enforcement author-

[37] For instance, with linear demand $q(p) = a - bp$, quadratic cost ($c(e) = \frac{1}{2}e^2$) and uniform distribution of cost, the function $\hat{e}(i)$ is monotone decreasing if, and only if,

$$\frac{a^2}{b} > \frac{-12(2\alpha - 3\beta)(-2\alpha+\beta)^2}{\beta^4}.$$

ity. Clearly competition law has implications beyond price, for instance on the formation of mergers; the structure of pricing (for instance, price maintenance, price discrimination issues, etc.); the tying of retailers; and so forth. Having disposed of the relatively simple pure price regulation case first, we leave this topic to future research.

## 7.6 Conclusion

This paper has examined the restrictions placed on the action space of an international competition authority that interacts with a set of subsidiary national industry regulators. We have isolated an important limiting factor on the set of actions an international competition authority (in its price regulating function) would wish to take. This limiting factor is the reciprocal externality the international authority imposes by increasing its investigation intensity: the more closely the agency monitors an industry (and therefore the more often it will overrule the regulator's decision with its own), the lower the incentive for the industry regulator also to invest effort into regulation of the industry. But this has implications for the international authority. Lower regulatory effort in general has two implications for the international authority: First, lower regulatory effort implies that the monopoly firm's pricing decision prevails relatively more often. But secondly, and this is the effect this paper has sought to isolate, lower regulatory effort may imply that less information is available to the international authority on which to base its pricing decision. The mechanism we have exploited in this paper is the observation that, under European Union antitrust law, the European Commission's (the international authority's) powers of investigation are clearly subordinate to the member state authorities' powers. The modelling tool we have used is to view the information available to the international authority as a subset of the information obtained by the regulator. This approach, while clearly extreme, allows us to focus more clearly on the nature of the bilateral externality between international authority and industry regulator.

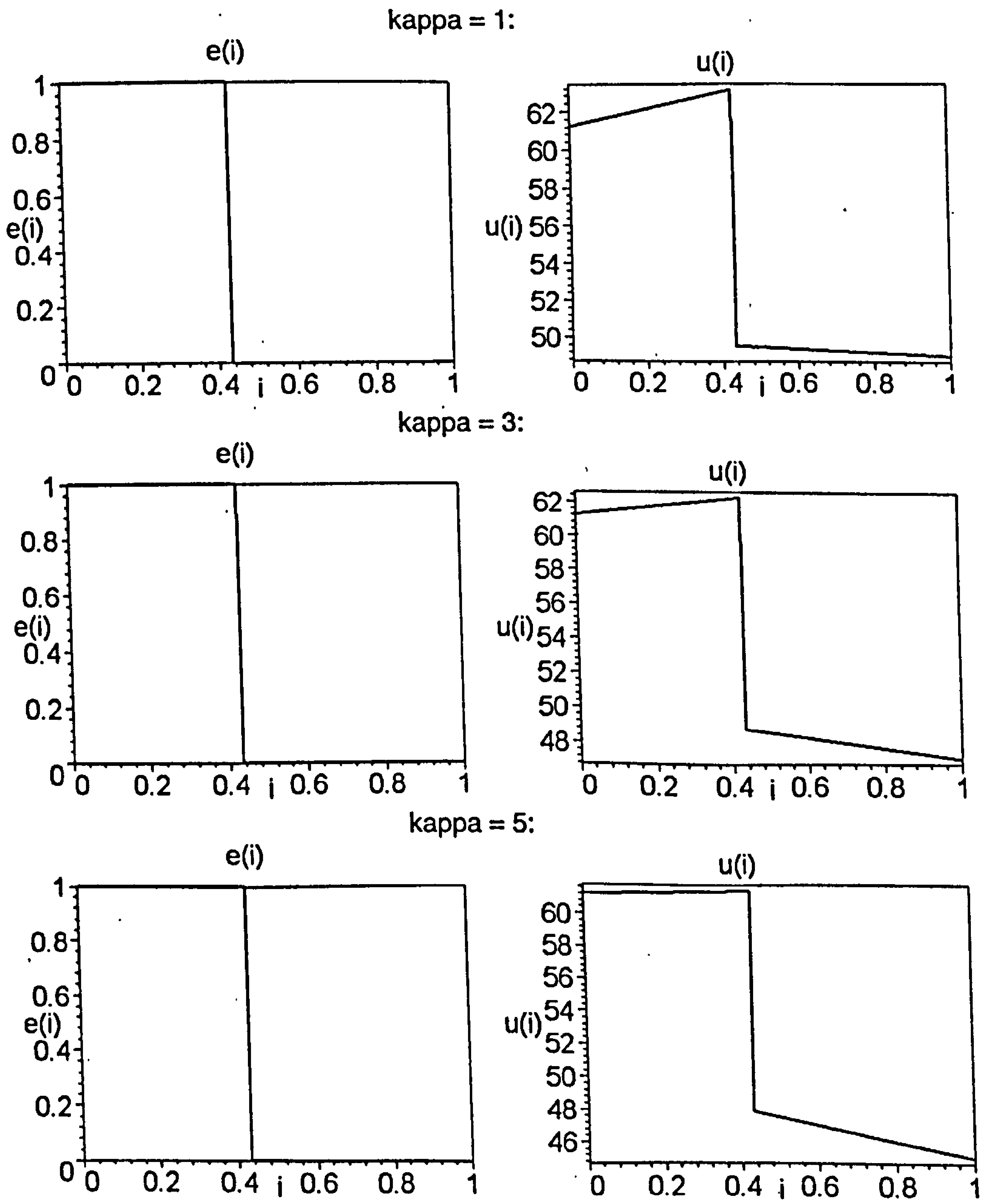
The most general result from our model is that international authorities have to be wary of overregulation: the more the international authority investigates, the more the regulator aligns itself with the interests of the regulated firm. This is a novel twist on the old regulatory capture hypothesis.

More specifically, the results we obtain from our model are limiting results. We can derive bounds on the international authority's actions that should be observed. In particular, we predict that regulatory effort will never be completely crowded out by international authority investigation. We obtain a surprisingly sharp limiting result in a special case, that predicts that regulatory effort is never crowded out by more than 50%. We also have a result that suggests that international authorities should either do nothing or investigate with some degree of rigor: dabbling, our model suggests, under a wide range of circumstances, is not optimal.

The results from the present paper should give material for thought to policy designers and economists alike. To policy designers who set the budget for international authorities, the advice is that more is not always better. To the economist, a note of caution: there is much that we still have to learn about the interaction of authorities in hierarchical structures that compete for policy outcomes.

## 7.7 Appendix

This appendix contains the numerical examples referred to in section 7.5.



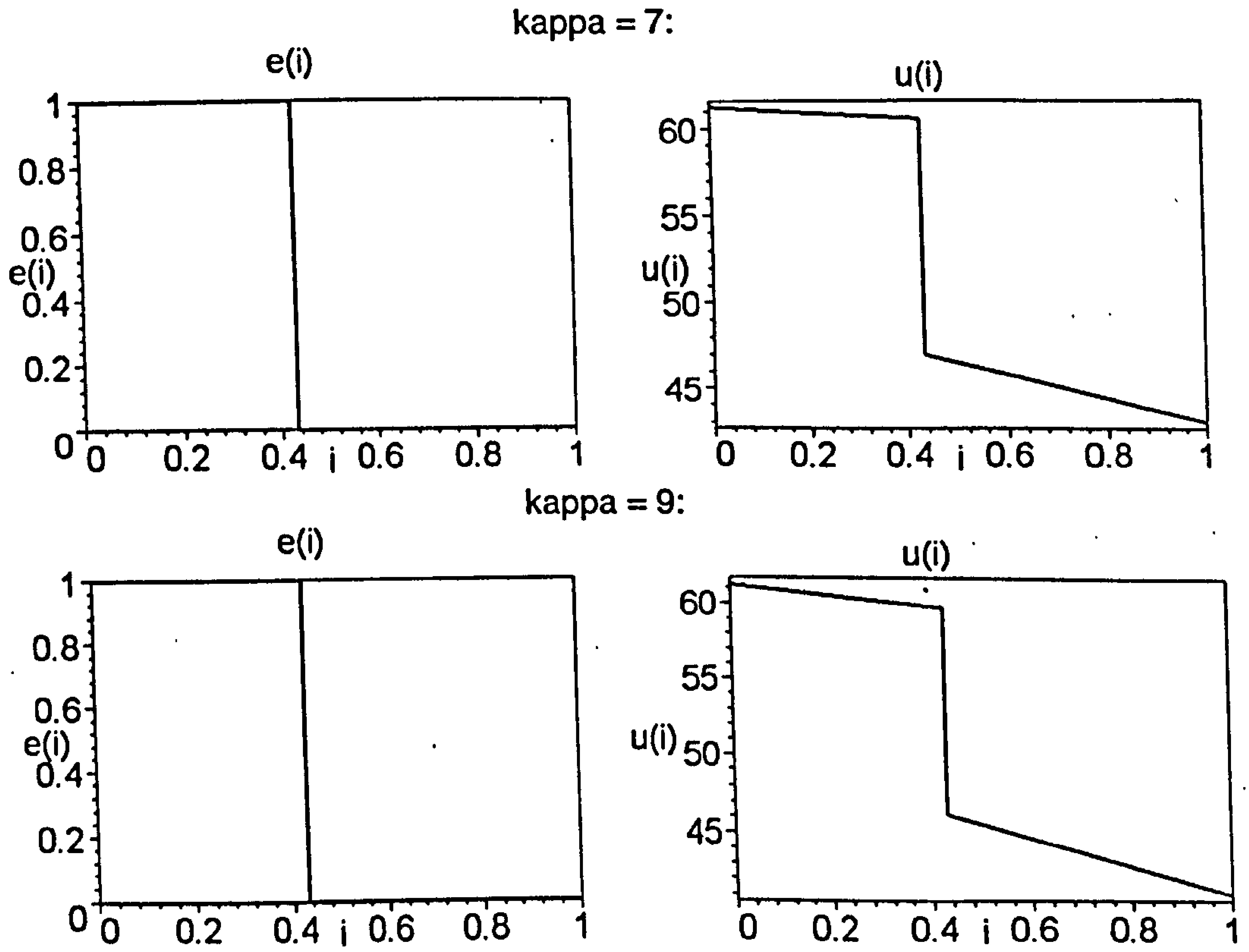
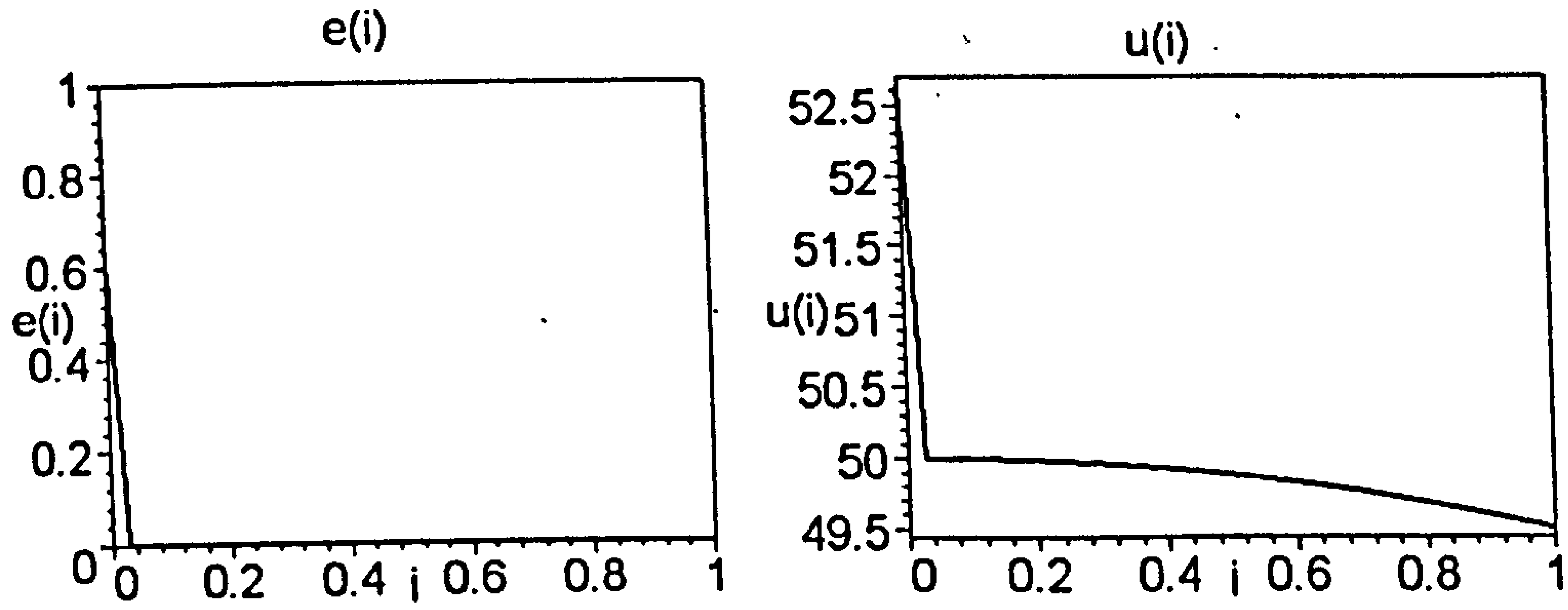
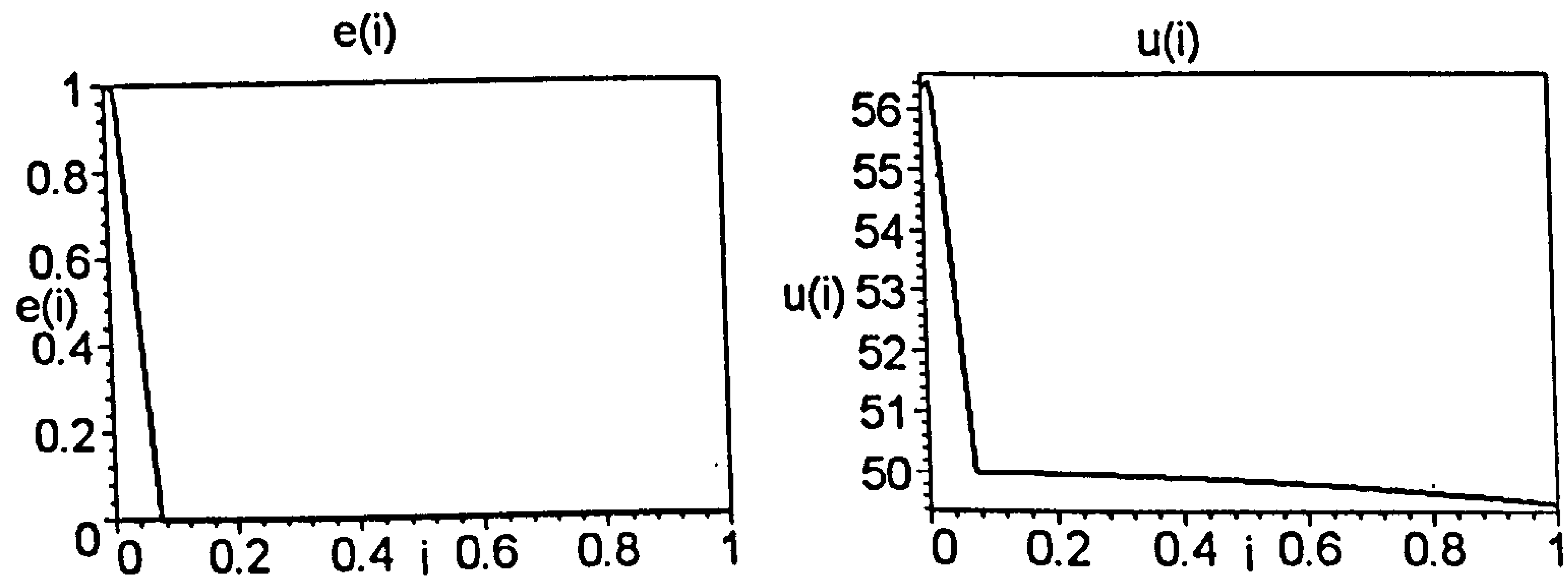



Figure 7.1: Numerical Example: $e(i)$, $u(i)$ for Parameter Values $a = 20$, $b = 1$, $\alpha = 1$, $\beta = 0.5$, $k = 1$

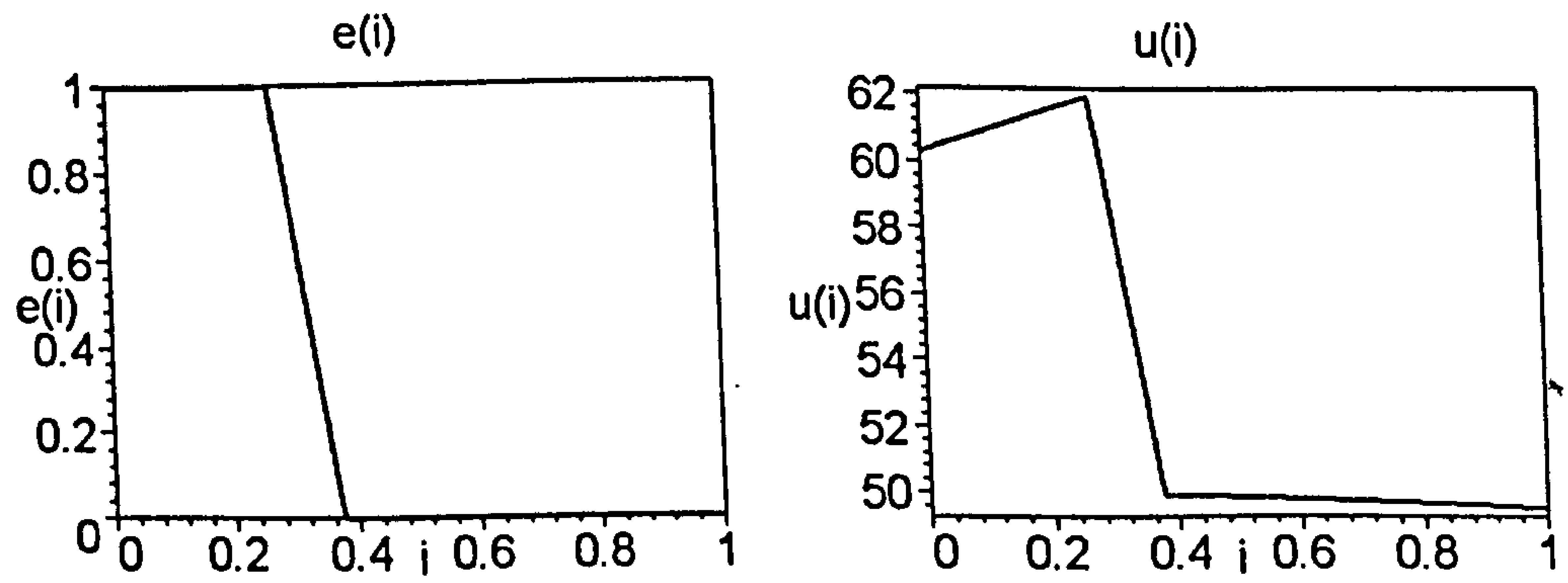
beta = .25:




beta = .35:




beta = .55:



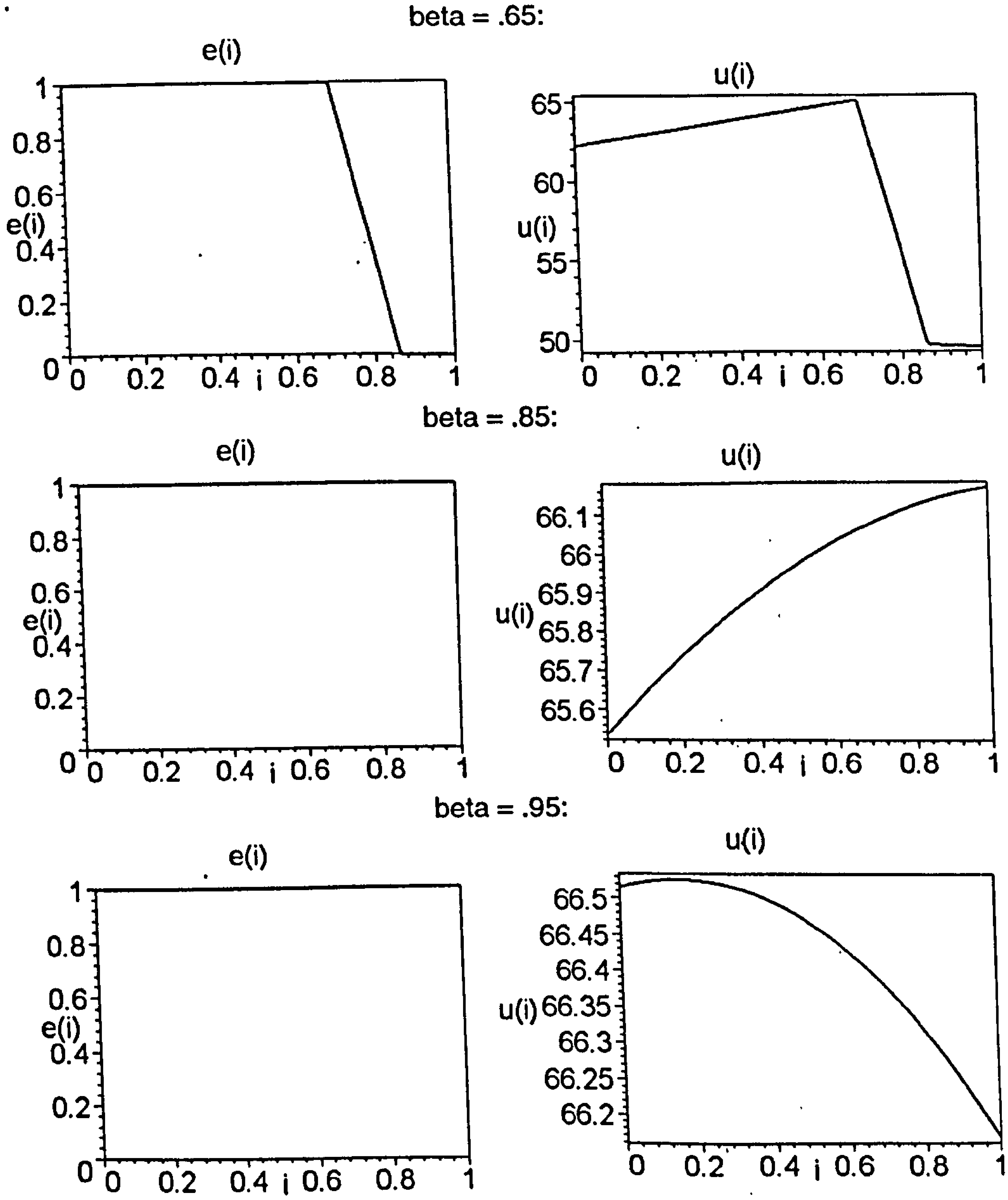



Figure 7.2: Numerical Example: $e(i), u(i)$ for Parameter Values $a = 20$, $b = 1$, $\alpha = 1$

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